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# Validation of Theoretical Coning Models Using Numerical Single-Well Simulation

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# صلاحية النموذج النظري لتشكل مخروط الماء باستخدام تشبيه رقمي لبئر فردية

سالم الأخضر وعبد الحميد وريث والمبروك بن روين

يعتبر تشكل مخروط الماء من المشاكل التشغيلية الحادة في المكامن النفطية المنتجة بقوة دفع الماء التحتي. وفي ليبيا، يوجد عدد كبير من الحقول النفطية التي تعاني من ظاهرة تشكل مخروط الماء. ويراعي عند الانتاج، من هذه الحقول، الحد الأدنى (الحد الحرج) لمنع فرصة تشكل مخروط الماء من الحدوث. ونظراً لأن مثل هذه المعدلات تعتبر ضئيلة (وغير إقتصادية)، فإن الشركات عادة، لتفادي مثل هذه الظاهرة، ما تبقى على معدلات الانتاج العادية، وفي هذه الحالة يصبح من الضروري تقدير زمن وصول المخروط المائي لخرم الانتاج السفلية (زمن الاختراق) وذلك من خلال إعداد تصور لسلوك الماء المستقبلي الكفيل بإعداد التصاميم المناسبة لتجهيزات الانتاج السطحية.

لقد تم تطبيق بعض المقارنات المنشورة في المسائل التي تناولت توقعات الحد الحرج للإنتاج وزمن الاختراق وسلوك نسبة الماء، وذلك لمعلومات فعلية من ثلاثة آبار ليبية.

كما تم إنشاء نموذج لبئر ثنائي الأبعاد (2D) وبشكل شعاعي لآبار مختارة، وقورنت معلومات هذا النموذج مع معلومات الأداء الفعلية ونتائج المقارنات.

وبالتأكيد، فإن هذه الدراسة أوضحت أن بعض المقارنات الرقمية لها مصداقية أكثر من البعض الآخر. كما أن الدراسة تقر بإمكانية وضع سلوك تشبيهي لتشكل المخروط المائي لإعطاء نتائج أفضل، إلا أن قلة وقصور المعلومات المتعلقة بوصف المكمن والنفاذية العمومية، وعموماً قصور معلومات تحاليل العينات اللبية الخاصة وكذلك التوثيق، تساهم في تدني كفاءة النتائج المقتبسة من النماذج الرقمية، وعلى الأخص في الأحيان المتأخرة.

وأخيراً، تستهدف هذه الدراسة وضع إجراءات مناسبة لتحليل ظاهرة تشكل مخروط الماء في المكامن. وبالرغم من أن الدراسة اعتمدت على معلومات فعلية لثلاثة آبار، إلا أن فكرة توسيعها لعدد أكبر من الآبار والطبقات مازال قائماً، وذلك بهدف إثبات صحة نتائج النماذج من عدمه.

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**Abstract:** *Water coning is often a serious operational problem in oil reservoirs producing with strong bottom water drive. In Libya, many of the oil fields have water coning problem. The production from such fields would normally consider limitation of the production rate to a certain minimum (critical rate) to prevent water cone from happening. However, since such a rate is normally very small (i.e. not economical), the company will be producing at a higher rate, and it becomes essential to estimate the time required for the water cone to reach the lower perforations (i.e. breakthrough time), and to predict the water*

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*performance afterwards to make the proper design for the surface facilities.*

*Some of the published correlations for predicting the critical oil rate, breakthrough time, and water cut performance were applied to actual well data from three Libyan wells. Also, a single well 2D radial model was constructed on selected wells. The model results were compared to the actual field performance as well as to the correlations results.*

*This study shows clearly that some of the empirical correlations can be considered more reliable than the others. It also, shows that even though simulation of water coning is possible, and give more reliable results, however, the lack of data (i.e.good reservoir description, lack of vertical permeability data on a layer basis, lack of a reliable special core analysis, and improper documentation of work-overs) will always limit the validity of the coning results obtained by the numerical models especially at late times.*

*Finally, this study is mainly aimed to set up the proper procedure of analyzing the water coning phenomena. Since the study considers the actual field data of only three wells, it should be expanded to a larger number of wells producing from different formations to confirm (or disconfirm) the results and conclusions of this study.*

## INTRODUCTION

Water coning is a serious problem in many oil field applications. It is the mechanism responsible for the rise of bottom water through the oil zone to the producing interval. The production of coned water will reduce oil production significantly. Therefore, accurate estimate of the break-through time and the water cut performance afterwards is of extreme importance in reservoir development and management.

### Theoretical and Experimental Literature Review:

Many authèrs have offered solutions for the steady state watèr coning problem. The first of these was presented by Muskat[1], in 1937. He presented an approximate solution based on many assumptions such as, single-phase (oil) potential distribution around the well at a steady-state conditions which is described by the solution of Laplace's equation for incompressible fluid; uniform flux boundary condition at the well, and the potential distribution in the oil phase is not influenced by the cone shape.

Arthur[2] developed a set of charts to solve the water coning problem, based upon Muskat's treatment. He showed that the vertical thickness of the two fluids below the wellbore is very important, with coning occuring with decreasingly smaller drawdowns, as the distance between the well and the coning fluid shortens. Analysis of his charts lead to the conclusion that coning is greatly restricted by small lenses of reatively low permeability directly below the bottom of the well, since they greatly distort the potential gradients in the sand.

Chaney *et al.*[3] developed a set of curves from which critical flow rates can be determined at various lengths of perforations. This method is an extension of Muskat's work and based upon the results of mathematical and potentiometric analysis of water coning.

Chierici *et al.*[4] utilized a large potentiometric model of the electrolyte tank type, which was used to determine the maximum water-free oil production rate for a given perforated interval.

Meyer and Garder[5] analytically determined the maximum allowable flow rate of oil into a well without water zone coning into the production section of the well.

Schols[6] presented an emperical critical rate correlation for partially penetrating wells. This relation is based on laboratory experiments and supplemented by a number of mathematical simulations. In Hele-shaw model, the steady state flow of segregated fluids in porous media in a two dimensional, radial flow configuration is simulated.

Chaperon[7] provided a simple and practical estimate of the critical rate to be expected without water zone coning into the production section of the well. This estimate is based on a numerical model results.

Sobocinki and Cornelius[8] give an emperical correlation for predicting water coning breakthrough time based on laboratory data and computer program results. Their experimental laboratory studies were conducted with a pie-shaped, sand-packed plexiglass model. The correlation involves dimensionless groups of reservoir and fluid properties, production rate, and well characteristics.

Bournazel and Jeanson[9] developed a new method combining experimental correlations using dimensionless numbers for estimating breakthrough

time with a simplified analytical approach based on the assumptions that the front shape behaves like a current line, in an equivalent model of different shape. Conversely, this method can be used for approximately determining the optimum completion and withdrawal rate.

Kuo and DesBriasy[10] presented a correlation for the prediction of water cut performance in a bottom water drive reservoir. Numerical simulation was used to determine the sensitivity of water coning behaviour to various reservoir parameters. They published an equation that accurately reproduces the graphical water coning breakthrough time results of Sobocinki and Cornelius[8]. It is important to note that this correlation was developed using straight line relative permeability curves.

## MODEL DESCRIPTION

The data used in the analysis included actual field data from three Libyan oil wells (wells A-06 and A-21/ Bahi Field-Courtesy of Waha Oil Company, Tripoli, Libya, and well B-15/ Sharara Field-Courtesy of REPSOL, Tripoli, Libya).

Average horizontal permeability was obtained from pressure build-up and productivity index tests. The horizontal permeability distribution for each layer in the model was determined from core data.

As a starting point, the initial vertical permeability values were obtained from plots of horizontal versus vertical permeability from core data.

Individual well production rates (oil and water) were collected. Whenever the production rates were realtively constant, the model used an average rate over these time periods to minimize calculation times.

Since there was no data provided for the aquifer thickness except that it is a very thick aquifer, consequently, the single well models used an aquifer thickness of 1000 feet.

All the required well data, rock properties per layer, fluid properties, and special core analysis data for the three different wells are shown in Tables A3-A6 in appendix-A.

### Model-1 (Well A-06)

This model consists of 10x1x21 homogeneously parameterized radial gridblocks. This well is in the south-central part of Bahi Field. The well was put on production in April 1970. The pay zone is relatively thick (95 ft). The present water cut has reached 60%. Thickness of the vertical layers for the model and the initial oil-water contact were obtained from well logs.

### Model-2 (Well A-21)

This model consists of 10x1x15 homogeneously parameterized radial gridblocks. This well is in the western area of Bahi Field. The well was put on production in February 1970. The water cut rose very sharply from 0 to 35% during the first year of production. The pay zone is reatively thick (95 ft).

### Model-3 (Well B-15)

This model consists of 10x1x15 homogeneously parameterized radial gridblocks. This well is in the east-central part of Elsharara Field. The well was put on production in April 1998. The water cut increase was very slow during the first year of production. Then, the water cut had a sharp increase from 0.07 to 36% afterwards.

## RESULTS

### 1. Theoretical Correlations:

#### *Critical Rate Correlations*

The results obtained for wells A-06, A-21, and B-15 using Meyer and Garder[6], Chaney *et al.*[3], Schols[6], and Chaperon[7] correlations are shown in Table 1. The results show that the critical rates vary widely for each well. However, in all cases, the calculated critical rates are very low and would be un-economical for Libyan fields.

**Table 1. Critical oil flow rates**

| Theoretical Correlations | Critical Oil Rate, BPD | | |
|---|---|---|---|
| | Well: A-06 | Well: A-21 | Well: B-15 |
| Meyer and Garder | 12 | 2 | 26 |
| Chaney, Nable, Henson and Rice | 44 | 6 | 83 |
| Schols | 23 | 3 | 49 |
| Chaperon | 5 | 0 | 51 |

#### *Breakthrough Time Correlations*

The two correlations used are the Sobocinki and Cornelius[8], and Bournazel and Jeanson[9] . The results are shown in Table 2.

Table 2. Breakthrough time

| Theoretical Correlations | Breakthrough time (Days) | | |
|---|---|---|---|
| | Well: A-06 | Well: A-21 | Well: B-15 |
| Sobocinki and Cornelius | 1697 | 305 | 1458 |
| Bournazel and Jeanson | 750 | 150 | 600 |

The Breakthrough time plots representing well A-06, A-21, and B-15 are shown in figures 1, 2, and 3 in appendix-B.

Two major conclusions can be easily drawn from these plots; first that Bournazel and Jeanson[9] correlation give normally a good estimate for the actual Breakthrough time as can seen from figures 1 and 3. Second, that the breakthrough time estimated by Sobocinki and Cornelius[8] correlation is very optimistic (at least twice the actual time) and consequently should not be depended on in production forecasts.

### *Water Cut Correlation*

Kuo and DesBrisay[10] developed simple equation for water cut performance prediction, and simple material balance equations were used to predict the location of the oil-water contact. This correlation was compared to the numerical results of water cut performance after breakthrough for the three wells as shown in figures 4, 5, and 6 in appendix-B.

Kuo and DesBrisay[10] gives a good water cut performance prediction for well A-06 from the time of breakthrough (2 years) to more than 10 years. Afterwards, the correlation would be inaccurate due to systematic shut-downs (causing water cone to move down), lowering of production rates, and workovers which prohibit the use of the correlation effectively.

The correlation also gives a good water cut prediction for well A-21 for the first

three years of production, Afterwards, long periods of shut-downs and reduced rates prohibit the effective used of the correlation.

Finally, the corrrelation also gives a good water cut performance prediction for well B-15 throughout the short history of the well. The water cut increase trend is maintained by the correlation inspit of the average difference of < 20% between the actual data and the correlation estimates.

Consequently, it can be concluded that Kuo and DesBrisay[10] give in general a good prediction of the water cut performance after breakthrough provided that minimum human interference in the form of reducing rates, prolonged shut-in periods, or re-preforating is applied to the oil well.

## 2. Single Well Model:

The 2D radial numerical model was constructed using Eclipse-100 black-oil simulator. Every well has different grid system depending on the geologic layers. Also, in the Z-direction, the number of cells for each well was varied with the objective of keep $\Delta Z$ as small as possible in order to minimize the effect of numerical dispersion if any.

Since the problem here is to study the vertical water movement (*i.e.* water coning), history matching was achieved by changing two main influencing parameters which are the Relative Permeability Curves, and the Absolute Permeability Values. Since Bournazel and Jeanson[9] correlation gave a good estimate for the actual breakthrough time, it was decided to maintain the end points of the relative permeability curves, and to restrict the changes to the shape of the curves for the three different wells. The horizontal permeabilities were first modified in order to obtain the actual flow rates reported for each well, then the vertical permeabilities for each layer were modified based on values calculated from core data.

### *Model-1 (well A-06)*

Several history match runs were made. The water cut history calculated by the model is compared to the actual water cut history. As can be seen from figures 7 and 8 in appendix-B, there is excellent agreement between the observed and simulated water cuts for the first 10 years. Afterwards, it was not possible to obtain a good history match (Fig. 8). Many factors could be causing such divergence: either the changes incorporated in the relative permeability curves are not accurate at high water saturations since the water curve increases exponentially in most cases, or because the input of the workovers was not accurate, or because the reducing and shutting of the well for a prolonged periods of time affects the wettability (and consequently the relative permeability curves) which can not be included in the model.

### *Model-2 (well A-21)*

Similar to the performance of well A-06, there is excellent agreement between the observed and

simulated water cuts for the first three years as shown in figure 9 in appendix-B. Afterwards, it was not possible to obtain a good history match (Fig. 10 in appendix-B). Same factors mentioned earlier for well A-06 could be causing divergence at late times. These factors include incorrect changes incorporated in the relative permeability curves, improper input of workovers, or the shutting of the well for a prolonged periods of time creating wettability changes which can not be included in the model.

*Model-3 (well B-15)*

After several history match runs, a good agreement between observed and simulated water cuts as shown in figures 11 and 12 in appendix-B. The average deviation of 20% in the calculated values is most properly due to incorrect changes in the end points of the relative permeability curves.

*Final Comparison*

A final comparison of the actual breakthrough time and water cut performance to the results calculated using the theoretical correlations and the simulated coning model is shown in figures 13, 14, and 15 in appendix-B for the three wells respectively.

The comparison clearly indicates the following:

- The single well numerical model would always give more reliable matching for water cut performance than the emperical correlations available in the literature.
- Some emperical correlations can be considered more reliable than the others. For example, good identification for the time of breakthrough can be obtained using Bournazel & Jeanson[9] correlation, whereas a good identification of the general trend of the water cut increase can be obtained using Kuo and DesBriasy[10] correlation.
- Production at very high oil rates with high water production rates can be problematic in predicting water cut performance by theoretical correlations as well as by single well simulation modeling.

## CONCLUSIONS AND RECOMMENDATIONS

### 1. Coning Correlations

The theoretical correlations discussed in this study were applied to three Libyan oil wells in order to rapidly evaluate the water coning performance parameters (*i.e.* critical rate, breakthrough time, and water cut performance after B.T). The main conclusions are:

*Critical Rate*

- The different mathematical correlations available in literature vary widely in their estimate of the critical rate value.
- The Chaney et al[3] correlation gives always the highest estimate for the critical rate.
- In all studied wells, and using any of the available mathematical correlations, the calculated critical rates will be very low and would be uneconomical for Libyan fields.

*Breakthrough Time*

- The Bournazel and Jeanson[9] correlation gives normally a good estimate for the actual breakthrough time, whereas, the Sobocinki and Cornelius[8] correlation would always give very optimistic results (at least twice the actual time) and consequently should not be applied in production forecasts.

*Water cut Performance*

- The Kuo and DesBrisay[10] correlation give, in general, good prediction of the water cut performance after breakthrough provided that minimum human interference in the form of reducing rates, prolonged shut-in periods, or re-perforating is applied to the oil well.

In general, the study clearly indicates that the use of the mathematical correlations should be limited to rough estimates of breakthrough time or water cut performance, in case of project costs or limited well data do not allow the more reliable single well modeling.

### 2. Coning Numerical Model

The main conclusions obtained from the single well simulation are:

- The history matching indicates a good match for the first period of production (until few years after breakthrough time or as long as the water cut is relatively small, *i.e.* > 50%).
- The single well simulation model was not able to give a good match for the water cut performance

at late times, in spite of all the changes made in the petrophysical properties (horizontal and vertical permeabilities), and in the shape and end points of the relative permeability curves.

• Most probably, the inability to obtain a good history match at late times was due to:

1 - Using a single set of relative permeability curves to represent the whole thickness of the oil zone. It is clear that to obtain a good match, it is necessary to have a set of relative permeability curves for each layer in the oil zone. Normally, this is not available for any one single well.

2 - The frequent shut-in of the wells for prolonged periods might have created changes in the wettability characteristics of the rock and most certainly have changed the shape of the relative permeability curve with time for each single layer.

In general, the single well numerical model would always give more reliable matching for the water cut performance than the empirical correlations available in the literature. However, reliability of the model will be highly dependent on quantity of data available (especially, good layer-by-layer description in terms of vertical and horizontal permeabilities and relative permeability curves). Also, even when a good history match is obtained, the validity of the model will be declining with time with continuing fluctuations in the production and frequent shutting down of the well.

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# Appendix-A

**Table A 3. Rock properties**

| | Well A-06 | Well A-21 | Well B-15 |
|---|---|---|---|
| Rock compressibility, $psi^{-1}$ | $5.1x10^{-6}$ | $5.1x10^{-6}$ | $4.3x10^{-6}$ |
| Porosity, fraction | 0.26 | 0.26 | 0.11 |

**Average porosity and permeability for individual layers:**

Well A-06

| Layer | Top s.s. | Base s.s. | Average porosity fraction | K md | h ft | md Kv |
|---|---|---|---|---|---|---|
| 1 | 2590 | 2615 | 0.260 | 6 | 25 | 4 |
| 2 | 2615 | 2639 | 0.263 | 16 | 24 | 6 |
| 3 | 2639 | 2663 | 0.278 | 44 | 24 | 11 |
| 4 | 2663 | 2705 | 0.267 | 62 | 22 | 10 |

Well A-21

| Layer | Top s.s. | Base s.s. | Average porosity fraction | K md | h ft | md Kv |
|---|---|---|---|---|---|---|
| 1 | 2625 | 2664 | 0.200 | 187 | 39 | 79 |
| 2 | 2664 | 2701 | 0.260 | 204 | 37 | 81 |
| 3 | 2701 | 2716 | 0.268 | 349 | 15 | 83 |

Well B-15

| Layer | Top s.s. | Base s.s. | Average porosity fraction | K md | h ft | md Kv |
|---|---|---|---|---|---|---|
| 1 | 2971 | 3031 | 0.11 | 1152 | 60 | 205 |

**Table A 4. Reservoir fluid properties**

**Oil properties:**

Well: A-06 and A-21

| P(psia) | Bo (RB/STB) | Rs(SCF/STB) | $m_o$(cp) |
|---|---|---|---|
| 2015 | 1.176 | 221 | 1.12 |
| 1315 | 1.182 | 221 | 1.06 |
| 1015 | 1.184 | 221 | 1.03 |
| 715 | 1.187 | 221 | 1.00 |
| 515 | 1.189 | 221 | 0.99 |
| 315 | 1.190 | 221 | 0.986 |
| 267 | 1.1909 | 221 | 0.98 |
| 231 | 1.191 | 213 | 1.00 |
| 189 | 1.183 | 202 | 1.02 |
| 141 | 1.177 | 189 | 1.05 |
| 97 | 1.166 | 170 | 1.07 |
| 72 | 1.159 | 155 | 1.09 |
| 58 | 1.151 | 142 | 1.12 |
| 49 | 1.144 | 130 | 1.42 |
| 15 | 1.051 | 0 | 1.86 |

Well: B-15

| P(psia) | Bo (RB/STB) | Rs(SCF/STB) | $m_o$(cp) |
|---|---|---|---|
| 1996 | 1.084 | 30 | 0.82 |
| 1570 | 1.088 | 30 | 0.79 |
| 1059 | 1.093 | 30 | 0.75 |
| 730 | 1.113 | 30 | 0.72 |
| 605 | 1.098 | 30 | 0.71 |
| 354 | 1.101 | 30 | 0.69 |
| 280 | 1.118 | 30 | 0.68 |
| 220 | 1.118 | 30 | 0.68 |
| 154 | 1.103 | 30 | 0.68 |
| 143 | 1.19 | 30 | 0.68 |

**Water properties:**

| | Well A-06 and A-21 | Well B-15 |
|---|---|---|
| Viscosity, cp | 0.54 | 0.54 |
| Compressibity, $psi^{-1}$ | $2.77x10^{-6}$ | $3.4x10^{-6}$ |
| Density, g/cc | 1.0284 | 1.0284 |

**Table A 5. Well data**

| | Well A-06 | Well A-21 | Well B-15 |
|---|---|---|---|
| Well type | Pumping | Pumping | Pumping |
| Total depth, ft (s.s.) | 5051 | 2716 | 5000 |
| Completed in layer | 1,2,3,4 | 1,2 | 1 |
| Driving mechanism | Bottom water | Bottom water | Bottom water |
| Skin | 0.0 | 0.0 | 0.0 |
| Starting production | 1/4/1970 | 1/2/1970 | 12/4/1998 |
| OBHP, psia | 1188 | 1188 | 1996 |
| OOWC, ft (s.s.) | 2699 | 2701 | 3048 |

**Table A 6. Saturation functions**

Well: A-06

| $S_w$ | $K_{rw}$ | $K_{ro}$ | $P_{cow}$ |
|---|---|---|---|
| 0.29 | 0.000 | 0.780 | 0 |
| 0.40 | 0.050 | 0.260 | 0 |
| 0.50 | 0.108 | 0.085 | 0 |
| 0.60 | 0.199 | 0.015 | 0 |
| 0.70 | 0.337 | 0.000 | 0 |
| 0.80 | 0.529 | 0.000 | 0 |
| 0.90 | 0.789 | 0.000 | 0 |
| 1 | 1.000 | 0.000 | 0 |

Appendix-A (*Cont.*)

Well: A-21

| $S_w$ | $K_{rw}$ | $K_{ro}$ | $P_{cow}$ |
|---|---|---|---|
| 0.36 | 0.000 | 0.800 | 0 |
| 0.46 | 0.040 | 0.38 | 0 |
| 0.50 | 0.065 | 0.250 | 0 |
| 0.55 | 0.090 | 0.165 | 0 |
| 0.60 | 0.120 | 0.100 | 0 |
| 0.65 | 0.165 | 0.065 | 0 |
| 0.70 | 0.210 | 0.030 | 0 |
| 0.80 | 0.350 | 0.005 | 0 |
| 0.9 | 0.620 | 0.000 | 0 |
| 1 | 1 | 0.000 | 0 |

Well: B-15

| $S_w$ | $K_{rw}$ | $K_{ro}$ | $P_{cow}$ |
|---|---|---|---|
| 0.110 | 0.000 | 1.000 | 0 |
| 0.125 | 0.000 | 1.000 | 0 |
| 0.140 | 0.000 | 1.000 | 0 |
| 0.188 | 0.008 | 0.467 | 0 |
| 0.261 | 0.018 | 0.265 | 0 |
| 0.356 | 0.042 | 0.120 | 0 |
| 0.453 | 0.080 | 0.048 | 0 |
| 0.548 | 0.132 | 0.016 | 0 |
| 0596 | 0.166 | 0.010 | 0 |
| 0.645 | 0.200 | 0.004 | 0 |
| 0.740 | 0.287 | 0.000 | 0 |
| 0.870 | 0.644 | 0.000 | 0 |
| 1.000 | 1.000 | 0.000 | 0 |

## Appendix-B

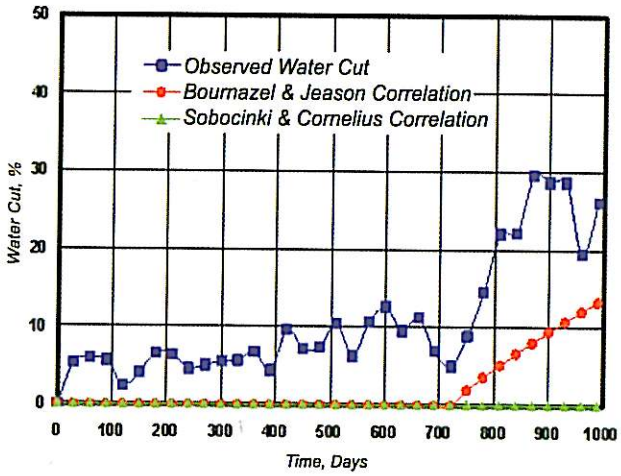



Fig. 1. Breakthrough time (theoretical correlations), well A-06.

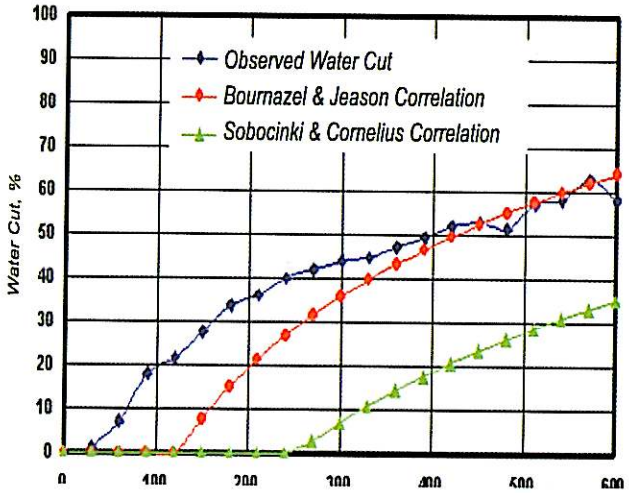



Fig. 2. Breakthrough time (theoretical correlations), well A-21.

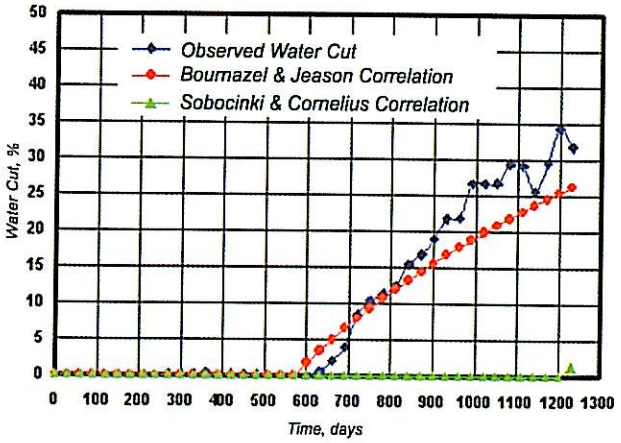



Fig. 3. Breakthrough time (theoretical correlations), well B-15.

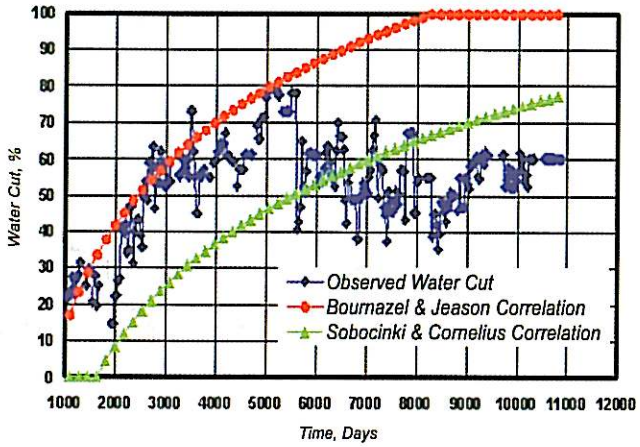



Fig. 4. Water cut performance using (Kuo and Desbrisay correlation) after breakthrough time calculated by Bournazel and Jeanson and Sobocinski and Cornelius correlarions, well A-06.

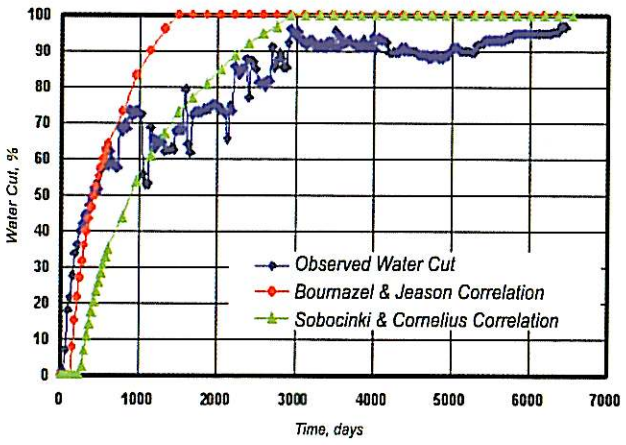



Fig. 5. Water cut performance using (Kuo and Desbrisay correlation) after breakthrough time calculated by Bournazel and Jeanson and Sobocinski and Cornelius correlarions, well A-21.

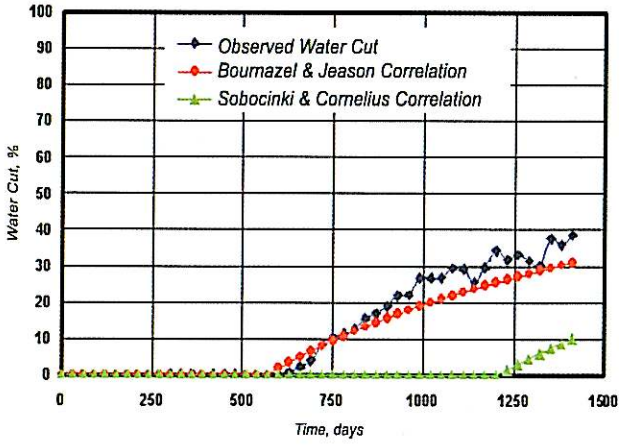



Fig. 6. Water cut performance using (Kuo and Desbrisay correlation) after breakthrough time calculated by Bournazel and Jeanson and Sobocinski and Cornelius correlarions, well B-15.

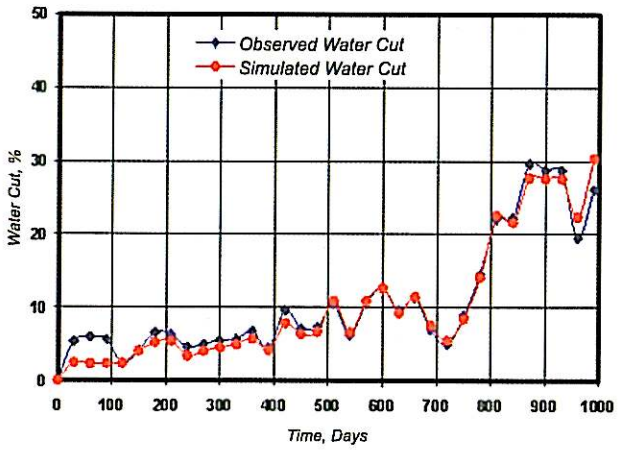



Fig. 7. Breakthrough time (simulation model), well A-06.

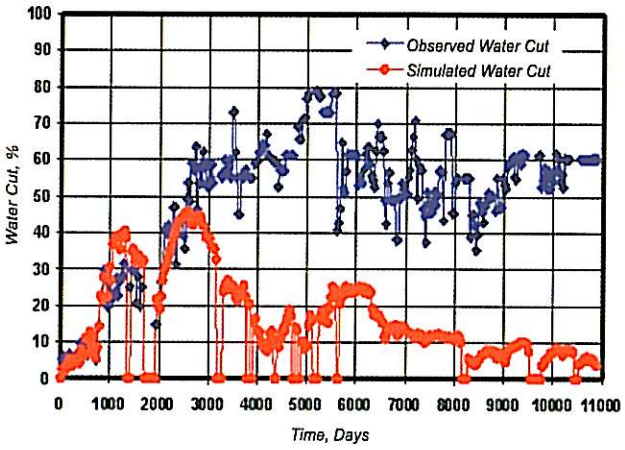



Fig. 8. Water cut performance after breakthrough, (simulation model), well A-06.

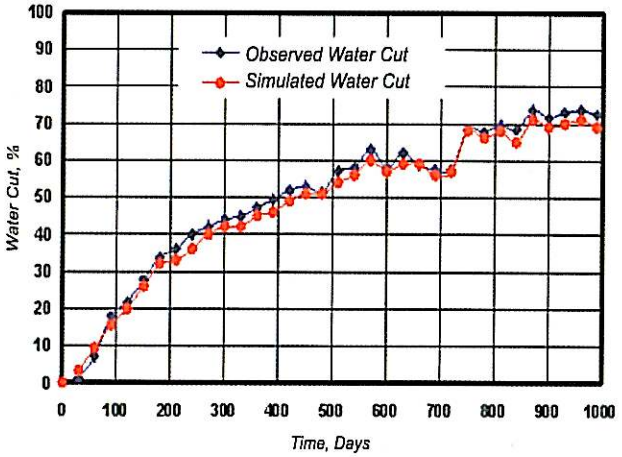



Fig. 9. Breakthrough time (simulation model), well A-21.

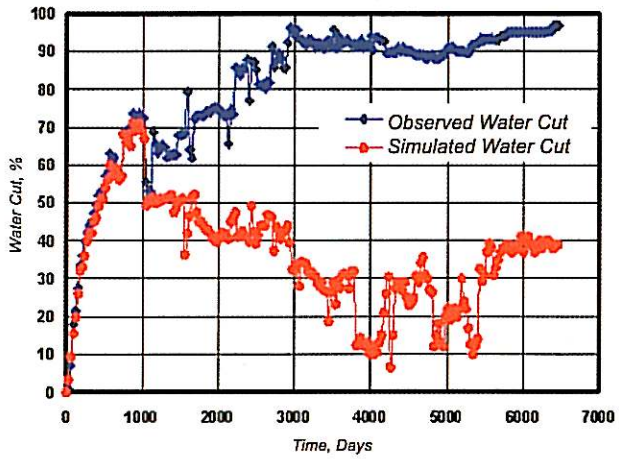



Fig. 10. Water cut performance after breakthrough, (simulation model), well A-21.

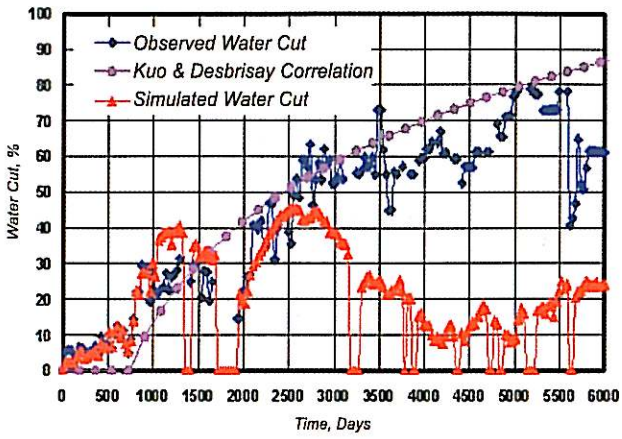



Fig. 13. Water cut performance prediction (theoretical and simulation model), well A-06.

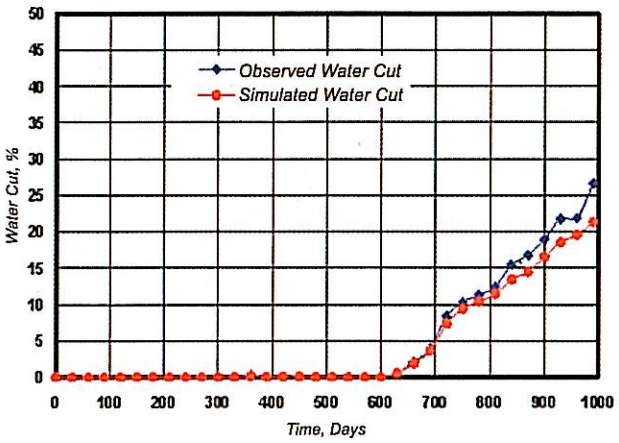



Fig. 11. Breakthrough time (simulation model), well B-15.

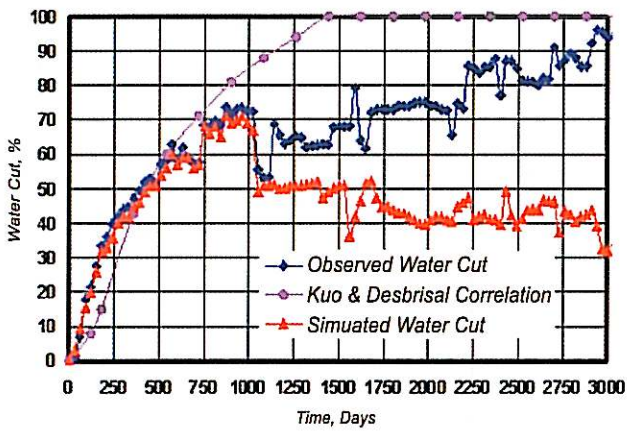



Fig. 14. Water cut performance prediction (theoretical and simulation model), well A-21.

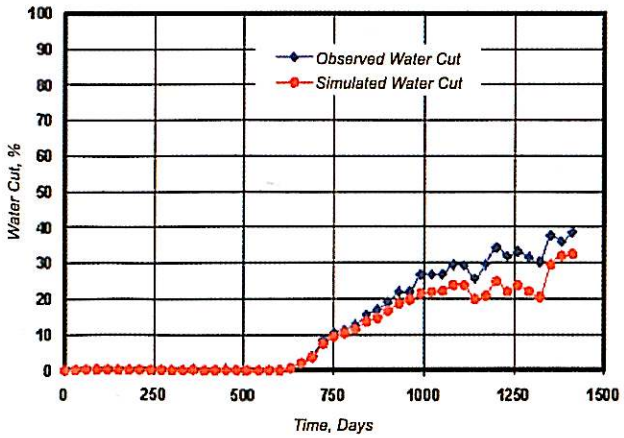



Fig. 12. Water cut performance after breakthrough, (simulation model), well B-15.

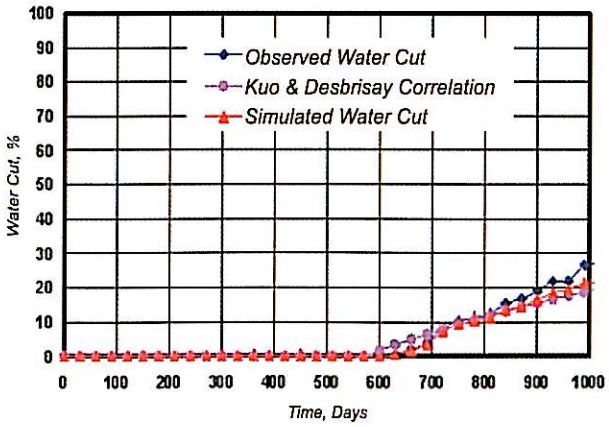



Fig. 15. Water cut performance prediction (theoretical and simulation model), well B-15.