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Three-Phase Oil Relative Permeability Models

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النفاذية النسبية ذات الأطوار الثلاثة للزيت

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نتيجة لصعوبة قياس النفاذية النسبية ذات الأطوار الثلاثة في المعمل، فإن معظم المهندسين يعتمدون على المعادلات النظرية لتوقع النفاذية النسبية للزيت. وقد تم في هذه الورقة مراجعة معظم المعادلات الرياضية المستخدمة عادة في توقعات النفاذية النسبية للزيت في حالة حركة عدة أطوار في الوسط المسامي. وكذلك مراجعة الدراسات المقارنة التي تمت بين المعادلات النظرية والتجارب العملية المنشورة في هذا الموضوع. وبصفة عامة يمكن القول بأن معظم المعادلات النظرية المتوفرة حاليا لإيجاد النفاذية النسبية للزيت لا تتوافق جيدا مع التجارب المعملية، وبأن الحاجة ما زالت ماسة لإجراء المزيد من الدراسات خاصة المعملية والتي من شأنها تحسين أداء المعادلات النظرية.

Abstract: *Due to the complexity of the three-phase relative permeability experiments, most of the engineers rely on the theoretical models to predict three phase oil relative permeability.*

In this paper, the three-phase oil relative permeability models that are commonly used for dealing with multiphase flow in porous media are reviewed. Comparison studies between the models and observed experimental oil relative permeability data published in the literature are also reviewed.

In general, most of the published models were not capable of producing a good match with the experimental data, and henceforth, more work is required in this area with particular attention to the experimental data needed to verify the theoretical models.

INTRODUCTION

Fluid flow in hydrocarbon reservoirs involves flow of oil, water and gas phases. Three-phase flow may take place in oil reservoirs producing below the saturation pressure, in retrograde gas-condensate reservoirs and in reservoirs under depressurization. Knowledge of three-phase relative permeability is essential to the studies of many types of enhanced oil recovery schemes such as CO_2 injection, water and gas flooding, steam flooding, gas injection, etc.

Three-phase relative permeability data are required to run reservoir simulation studies or to evaluate reservoir performance. These data can be obtained from laboratory measurements or by theoretical models. A number of experimental studies have been carried out since the pioneer work of Leveret and Lewis [9] in 1941. The difficulties observed during the experimental procedures and calculations have limited the work conducted in this area.Unfortunately, only limited amounts of published data are available in this subject. In the absence of reliable experimental data, many engineers have resorted to theoretical models. Theoretical models

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have made use of information such as two-phase relative permeability data, statistical models, and capillary pressure to predict three-phase relative permeability data.

The aim of this paper is to review the literature on three-phase oil relative permeability models that are commonly used for dealing with multiphase flow in porous media, as well as the comparison studies between the models and observed experimental oil relative permeability data [1-8].

THREE-PHASE OIL RELATIVE PERMEABILITY MODELS

Corey's[10] Model (1956)

The first model of relative permeability in three-phase flow was presented by Corey *et al* [10]. It is based on the concept of approximating the flow paths through the rock by the equivalent hydraulic radius of a bundle of capillary tubes. The phase permeability was assumed proportional to the mean hydraulic area or the square hydraulic radius. The relative permeability of the wetting phase and non wetting phase are dependent on their own saturation. The intermediate wetting phase (oil) occupies the flow channels which are intermediate in size between those occupied by water and gas. The model in its general form can be expressed by the following relationship:

$$K_{ro} = \frac{\left(s_l - s_w\right)^2 \int_{Sw}^{S_l} \frac{dS_l}{pc^2}}{\left(1 - s_{lr}\right)^2 \int_0^1 \frac{dS_l}{pc^2}}$$

For linear relationship between $\frac{1}{pc^2}$ versus liquid saturation S_l, the above equation can be simplified as shown in Table 1.

The model is applicable only for drainage type saturation history (oil displaced by gas) and requires only a single suite of gas/ oil relative permeability data measured at constant water saturation. The model can not be adjusted to force the end point of the isoperms to correspond to both measured two-phase oil/water relative permeability. The model was tested against the experimental data of Donaldson and Dean[7], and was found to give higher K_{ro} values than those obtained by Donaldson's and Dean, with larger discrepancy between two methods at low oil saturation.

Naar and Wygal [11] Model(1961):

This model [11] is based on the concept of flow in straight capillary tubes with a modification to allow blocking of the non wetting phase by the invading wetting phase. It was assumed that for the imbibitions oil permeability of consolidated sandstone the wetting and non wetting phase permeability depends on their saturation only while the intermediate phase depends on all three phase saturations. The model in its general form can be written as:

$$K_{ro} = \frac{\left[\frac{\left(1 - S^*_{wf}\right)}{\left(1 - S_{wc}\right)}\right]^3 S^*_{of} \int_0^{Sof} \left(S^*_{of} - S^*\right) \frac{dS^*}{pc^2}}{\int_0^1 \left(1 - S^*\right) \frac{dS^*}{pc^2}}$$

The model presented in Table 1 is for consolidated sandstones having a relationship of $\frac{1}{pc^2}$ versus S^* (effective saturation).

There is no method for adjusting the curve end points to fit either gas/oil data at irreducible water saturation or water / oil data at zero gas saturation. Manjnath and Hanarpour [12] compared the model with the experimental data of Owens and Archer [13].

The discrepancy between the two methods is evident at low oil saturation values. There is evidence of K_{ro} values depending only on S_o values especially low oil saturation in Naar and Wygal [11] model.

Land's [14] Model (1968):

Land [14] presented the model as a set of integral equations giving the dependence of relative permeability on saturation, saturation history and the relation between saturation and capillary pressure. The relative permeability can be obtained by properly selecting two rock properties: the residual non wetting phase saturation after the complete imbibitions cycle, and capillary pressure data .It was assumed that the maximum residual hydrocarbon saturation was the same, regardless of whether the initial hydrocarbon saturation is gas, oil or both. The model in its general form can be written for a relationship of $\frac{1}{pc^2}$ as a function of S^* .

Table 1. Chronological listing of three-phase oil relative permeability model.

DATE	AUTHOR	FORMULA
1956	Corey *et al*	$K_{ro} = \left[\frac{\left(S_L - S_w\right)^3}{\left(1 - S_{lr}\right)^4}\right] - \left[\left(S_w + S_l - 2S_{lr}\right)\right]$
1961	Naar and Wygal	$K_{ro} = \overset{*}{S}_{of}\left[\overset{*}{S}_{of} + 3\overset{*}{S}_{fw}\right]$
1968	Land	$K_{ro} = \left(S'_{of}\right)^3 * \left(2S'_w + S'_{of}\right)$ $K_{ro} = \left(S'_{of}\right) * \left(2S'_w + S'_{of}\right) - \left(S'_{of}\right)^2 * \left[\left(S'_{gr}\right)^2 + \left(\frac{2}{C}\right)\left(S'_{gr} + \frac{1}{C}\ln\frac{S'_{gr}}{S'_{gi}}\right)\right]$
1970	Stone I	$K_{ro} = \overset{*}{S}_o \beta_w \beta_g$
1973	Stone II	$K_{ro} = \left[\left(K_{row} + K_{rwo}\right)\left(K_{rog} + K_{rgo}\right) - \left(K_{rwo} + K_{rgo}\right)\right]$
1976	Dietrich and Bonder	$K_{ro} = \left[\frac{\left(K_{row} + K_{rwo}\right)\left(K_{rog} + K_{rgo}\right)}{K_{rocw}}\right] - \left(K_{rwo} + K_{rgo}\right)$
1976	Hirasaki	$K_{ro} = \left[\left(K_{row}K_{rog}\right) - S_g\left(1 - K_{row}\right)\left(1 - K_{rog}\right)\right]$
1977	Nolen	$K_{ro} = K_{rocw}\left[\left(\frac{K_{row}}{K_{rocw} + K_{rwo}}\right)\left(\frac{K_{rog}}{K_{rocw} + K_{rgo}}\right) - \left(K_{rwo} + K_{rgo}\right)\right]$
1979	Aziz and Settari	$K_{ro} = \left[\frac{\overset{*}{S}_o\left(K_{row}K_{rog}\right)}{K_{rocw}\left(1 - \overset{*}{S}_w\right)\left(1 - \overset{*}{S}_g\right)}\right]$ $K_{ro} = \left[\left(K_{row} + \omega K_{rwo}\right)\left(K_{rog} + \omega K_{rgo}\right) - \omega\left(K_{rwo} + K_{rgo}\right)\right]$
1984	Aleman	$K_{ro} = K_{ro}\left(I\right)^{-\Delta}$
1987	Parker *et al*	$K_{ro} = \left(\bar{S}_o\right)^{0.5} * \left[\left\{1 - \left(\bar{S}_w\right)^{\frac{1}{\lambda}}\right]^{\lambda} - \left\{1 - \left(\bar{S}_l\right)^{\frac{1}{\lambda}}\right]^{\lambda}\right]^2$
1989	Baker	$K_{ro} = \frac{\left[K_{row}\left(S_w - S_{wr}\right) + K_{rog}\left(S_g - S_{gr}\right)\right]}{\left[\left(S_w - S_{wr}\right) + \left(S_g - S_{gr}\right)\right]}$
1989	Pope	$K_{ro} = K_{rocw}\left[\alpha\left(\bar{S}_o\right)^{\beta}\left(1 - \bar{S}_w\right)^{\eta} + \left(1 - \alpha\right)\left(\bar{S}_o\right)^{\sigma}\left(1 - \bar{S}_g\right)^{\delta}\right]$

Table 1. ***Cont.***

1989	Lake	$K_{ro} = K^{\circ}_{ro}\left[\frac{\left(S_o - S'_{or}\right)}{\left(1 - S_{wr} - S_{gr} - S'_{or}\right)}\right]^{eo}$
1989	Corey-type	$K_{ro} = K^{\circ}_{ro}\left[\frac{\left(S_o - S'_{or}\right)}{\left(1 - S_{wr} - S_{gr} - S'_{or}\right)}\right]^{eo}$
1991	Marek *et al*	$K_{ro} = K_{row} S_{oew}\left[\frac{S_{wf}}{S_{wf} + S_g}\right] + K_{rog} S_{oeg}\left[\frac{S_g}{S_{wf} + S_g}\right]$
1999	Moulu	$K_{ro} = K_{ro}\left(2ph\right)\left[\left(S_l\right)^{\frac{4-D_L}{2-D_L}} - \left(S_w + S_{or}\right)^{\frac{4-D_L}{2-D_L}}\right]$ $K_{ro} = \left(1-m\right)\left(S_o^{\phi} - S_{oi}^{\phi}\right) + mK_{ro}\left[S_l^{\phi} - \left(S_w + Sor\right)^{\phi}\right]$
1999	Blunt *et al*	$K_{ro} = \frac{\alpha'\left[\left(S_w - S_{wi}\right)\left\{a_{\circ} K_{row}\left(S_{ofb}\right) + b_{\circ} K_{rgw}\left(S_{ofb}\right)\right\}\right]}{\left(S_w - S_{wi}\right) + \left(S_g - S_{gr}\right)}$ $+ \frac{\alpha'\left[\left(S_g - S_{gr}\right)\left\{a_{\circ} K_{ob}\left(S_{ofb}\right) + b_{\circ} K_{rgw}\left(S_{ofb}\right)\right\}\right]}{\left(S_w - S_{wi}\right) + \left(S_g - S_{gr}\right)}$ $+ \alpha' K_{ol} S_{ol} + \beta'\left[\frac{S_o}{\left(S_o + S_g\right)}\right]\left[a_{\circ} K_{row} S_{hf} + b_{\circ} K_{rgw} S_{hf}\right]$

$$K_{ro} = \left(S'_{of}\right)^2\left[\left(1 - S'_{gf}\right)^2 - \left(S'_w + S'_{of}\right)^2 - \frac{2}{C} * X\right]$$

$$X = A + B - C$$

$$A = \ln\frac{\left(S^*_{hr}\right)_{\max} + \left\{1 - \left(S^*_{hr}\right)_{\max}\right\}S^*_{gf}}{\left(S_{hr}\right)_{\max} + \left\{1 - \left(S^*_{hr}\right)_{\max}\right\}S^*_{gi}}$$

$$B = \frac{1}{\left(S^*_{hr}\right)_{\max} + \left\{1 - \left(S^*_{hr}\right)_{\max}\right\}S^*_{gf}}$$

$$C = \frac{1}{\left(S^*_{hr}\right)_{\max} + \left\{1 - \left(S^*_{hr}\right)_{\max}\right\}S^*_{gi}}$$

A special case of the above equation is presented in Table 1. Due to the lack of experimental data; the authors could not evaluate the accuracy of the model. Baker[15] reported that this model offers the best approach to data for modeling relative permeability with the capillary bundle model. The model qualitatively predicts an increase in wetting phase relative permeability (at a given saturation) caused by trapping of the non wetting phase.

Stone's Models:

Stone's first model and second model are the most commonly used expressions for the prediction of three-phase oil relative permeability. Both are based on probability models and both assume that the relative permeability of the wetting and non wetting phase depends only on the saturation of the wetting and non wetting phases respectively.

Stone's[4] First Model (1970):

Stone[4] proposed the following form for his first model:

$$K_{ro} = S^*_0 \beta_w \beta_g$$

Where: β_w is chosen to be proportional to the relative permeability of oil in the presence of water at zero gas saturation, and β_g is chosen proportional to the relative permeability to oil in the presence of gas at the connate water saturation.

Table 2 shows the definitions of the stone's parameters. Stone compared this model with the experimental data of Corey *et al*[6], and Saraf and Fatt[16]. Agreement of predicted values is quite good especially for the higher values of oil permeability. At the lower values, the agreement was not quite as good. The formula is valid only if K_{row} at residual water saturation and K_{rog} at zero gas saturation happened to be unity and it is likely to be most in error at low oil relative permeability. The best value for the parameter S_{om} that occurs in the model is not well optimized.

Oak *et al*[17] compared his experimental results with Stone's first model. In the primary DDI, Stone's model predictions were good at low gas saturation but failed to predict correctly at high gas saturation. In the IID saturation history, Stone's first model was satisfactory.

Saraf *et al*[8] compared their experimental data with stone's first model, and reported that predicted values of relative permeability were lower, particularly in the low oil saturation range. In general, the model was not capable of predicting oil relative permeability that reasonably close to experimental results.

Stone's[5] Second Model (1973)

Stone[5] proposed another model for the prediction of three-phase oil relative permeability, independent of values. The expression for this model is:

$$K_{ro} = (K_{row} + K_{rwo})(K_{rog} + K_{rgo}) - (K_{rwo} + K_{rgo})$$

The above equation can not be valid at connate water saturation and zero gas saturation. The expression can be valid only if each of the independent variables happened to be unity. Stone's second model tends to under predict oil relative permeability. Stone suggested that his second method gave reasonable agreement with the data of Corey *et al*[6], and the data of Saraf and Fatt[16].

Saraf *et al*[8] reported that Stone's second model predictions were poor at low oil saturation and closer to experimental values at high saturation range. In general, as in the first model, stone's second model can not predict values that are reasonably close to experimental data for strongly water-wet system.

Van Spronsen[18] reported that oil isoperms predicted by Stone's second model are convex whereas his experimental data isoperms are concave.

Oak, *et al*[17] compared Stone's second model with their experimental results. They reported that the model failed to predict oil relative permeability at high gas saturation for primary DDI and IID saturation history. Oak[19] found that Stone's models predictions of three phase oil permeability are unsatisfactory.

Marek *et al*[20] compared Stone's second model as well as their model against their experimental data and reported that Stone's model under predicts the relative permeability at low oil saturation range and over predicts the values at high oil saturation.

MODIFIED FORMS OF STONE'S MODELS

Dietrich and Bonder[21] Modification (1976)

Dietrich and Bonder [21] adjusted Stone's second model by normalizing with K_{rocw} (K_{ro} measured at connate water saturation).

$$K_{ro} = \left[\frac{\left(K_{row} + K_{rwo}\right)\left(K_{rog} + K_{rgo}\right)}{\left(K_{rocw}\right)}\right] - \left(K_{rwo} + K_{rgo}\right)$$

They observed that during their work with Donaldson and Dean's[7] data, their model produced unrealistic values of K_{ro} unless K_{rocw} was greater or equal to a minimum value.

Hirasaki's [21] Modification(1976)

Dietrich and Bonder[21] also reported Hirasaki's modification of Stone's first model :

$$K_{ro} = (K_{row} + K_{rog}) - S_g\,(1\text{-}K_{row})(1\text{-}K_{rog})$$

Permeability was calculated relative to with no need to specify a minimum value for as in Stone's first model.

Nolen's Modification (1977)

Molina [22] presented Nolen's modification of Dietrich's equation:

$$K_{ro} = K_{rocw}\left[\left(\frac{K_{row}}{K_{rocw} + K_{rwo}}\right)\left(\frac{K_{rog}}{K_{rocw} + K_{rgo}}\right)\right] - \left(K_{rwo} + K_{rgo}\right)$$

Table 2. Definition of the Parameters.

$\dot{\mathbf{S}}_{\mathbf{of}} = \frac{\left(S_o - S_{ob}\right)}{\left(1 - S_{wi}\right)}; \dot{\mathbf{S}}_{\mathbf{fw}} = \left[\frac{\left(S_w - S_{wi}\right)}{\left(1 - S_{wi}\right)}\right] + S_{ob}; \mathbf{S}_{\mathbf{ob}} = \left(1 - S_{wi}\right)\int_0^{1-a} xdx; \mathbf{a} = \frac{\left(1 - S_w\right)}{\left(1 - S_{wi}\right)}$	Naar and Wygal Model
$\beta_{\mathbf{w}} = \frac{K_{row}}{\left(1 - S_w^*\right)}; \beta_{\mathbf{g}} = \frac{K_{rog}}{\left(1 - S_g^*\right)}; S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om}}; S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om}}; S_g^* = \frac{S_g}{1 - S_{wc} - S_{om}}; \psi = \frac{a' - \frac{S_{orw} - S_{org}}{1 - S_{wc} - S_{org}}}{1 - S_{wc} - S_{org}}$ $S_{om} = S_{orw} - S_g\left[\frac{S_{orw} - S_{org}}{1 - S_{wc} - S_{org}}\right] - \psi\left[S_g\left(1 - S_{wc} - S_{org}\right) - \left(S_g\right)^2\right]; K_{row} = \overset{\circ}{K}_{row}\left(\frac{S_o - S_{orw}}{1 - S_{wr} - S_{orw}}\right)^{e_{ow}}$ $K_{rog} = \overset{\circ}{K}_{rog}\left(\frac{1 - S_g - S_{lrg}}{1 - S_{lrg} - S_{gr}}\right)^{e_{og}}; K_{rwo} = \overset{\circ}{K}_{rw}\left(\frac{S_w - S_{wr}}{1 - S_{wr} - S_{orw}}\right)^{e_w}; K_{rgo} = \overset{\circ}{K}_{rg}\left(\frac{S_g - S_{gr}}{1 - S_{lrg} - S_{gr}}\right)^{e_g}$	Stone's Model
$\mathbf{S'}_{\mathbf{w}} = \frac{\left(S_w - S_{wc}\right)}{\left(1 - S_{wc}\right)}; \mathbf{C} = \frac{1}{\left(S'_{gr}\right)_{\max}} - 1$	Land's Model
$\boldsymbol{\omega} = \frac{K_{rocw}}{K}$	Aziz and Settari Model
$\Delta = \frac{S_o^*\left(K_{row}^* - 1\right)\left(K_{rog}^* - 1\right)\left[K_{row}^* K_{rog}^* - K_{rwo}^* K_{rgo}^*\right]}{\left[\left(K_{rgo}^* - 1\right)\left(K_{row}^* - K_{rwo}^*\right)\right] - \left[\left(K_{row}^* - 1\right)\left(K_{rgo}^* - K_{rog}^*\right)\right]}$	Almena's Model
$\boldsymbol{\lambda} = \left[1 - \left(1/n'\right)\right]$	Parker's Model
$\bar{\mathbf{S}}_{\mathbf{o}} = \frac{\left(1 - S_w - S_g - S_{or}\right)}{\left(1 - S_{wr} - S_{gr} - S_{or}\right)}; \bar{\mathbf{S}}_{\mathbf{w}} = \frac{\left(S_w - S_{wr}\right)}{\left(1 - S_{wr} - S_{gr} - S_{or}\right)}; \bar{\mathbf{S}}_{\mathbf{g}} = \frac{\left(S_g - S_{gr}\right)}{\left(1 - S_{wr} - S_{gr} - S_{or}\right)}$	Parker and Pope Model
$\overset{\circ}{\mathbf{K}}_{\mathbf{ro}} = \overset{\circ}{K}_{rg} + G\left(\overset{\circ}{K}_{rw} - \overset{\circ}{K}_{rg}\right); \mathbf{eo} = eg + G\left(ew - eg\right); \mathbf{S'}_{\mathbf{or}} = S_{gr} + G\left(S_{wr} - S_{gr}\right); \mathbf{G} = \left(S_g - S_{gr}\right)\frac{\left[1 - \left(S_w - S_{wr}\right)\right]}{\left[\left(S_g - S_{gr}\right) + \left(S_w - S_{wr}\right)\right]}$	Lake's Model
$\mathbf{S}_{\mathbf{oew}} = S_o + \left[\frac{\&_w S_{wf}^n S_g^{2-n}}{S_{wf} + S_g}\right]; \mathbf{S}_{\mathbf{oeg}} = S_o + \left[\frac{\&_g S_{wf}^n S_g^{2-n}}{S_{wf} + S_g}\right]$	Marek Model
$\mathbf{m} = \frac{\left(WI + 1\right)}{2}; \phi = \frac{\left(4 - D_L\right)}{\left(2 - D_L\right)} \quad S_l = S_o + S_w; \quad S_{or} = S_{orw}\left[1 - \frac{S_g}{1 - S_{wr} - S_{org}}\right] + S_{org}\left[1 - \left(1 - \frac{S_g}{1 - S_{wr} - S_{gr}}\right)\right]$	Moulu's Model
$\boldsymbol{\alpha}' = Max\left\{Min\left[\frac{\gamma_{go}}{\gamma_{go}^{crit}}, 1\right], 0\right\}; \boldsymbol{\beta}' = Max\left\{Min\left[\frac{\gamma_{go}^{crit} - \gamma_{go}}{\gamma_{go}^{crit}}, 1\right], 0\right\}; \mathbf{S}_{\mathbf{hf}} = \frac{1}{2}\left[\left(S_h - S_{hr}\right) + \sqrt{\left(S_h - S_{hr}\right)^2 + \frac{4}{Cr}\left(S_h - S_{hr}\right)}\right]$ $\mathbf{S}_{\mathbf{hr}} = \frac{S_h^{\max}}{1 + C_h S_h^{\max}}; \mathbf{S}_{\mathbf{of}} = Min\left[\frac{S_g S_o^*, S_o}{S_g^*}\right]; \mathbf{S}_{\mathbf{ofb}} = Min\left\{\left(S_o - S_{ol}\right), Max\left(S_{hf} - S_{gf} - S_{ol}, 0\right)\right\}$ $\mathbf{a}_{\mathbf{i}} = Max\left[Min\left\{\frac{\rho_i - \rho_{go}}{\Delta\rho_o}, 1\right\}, 0\right]; \mathbf{b}_{\mathbf{i}} = Max\left[Min\left\{\frac{\rho_o o - \rho_i}{\Delta\rho_o}, 1\right\}, 0\right]; \mathbf{S}_{\mathbf{o}} = S_{ob} + S_{ol}; \mathbf{S}_{\mathbf{ol}} = Min\left[\frac{S_g S_o^*}{S_g^*}, S_o\right]$ $\mathbf{K}_{\mathbf{ro}} = \frac{K_{rg} K_{ro}^*(g)}{K_{rg}^*(o) S_{ol}^{*2}} S_{ol}^2; \mathbf{K}_{\mathbf{ob}}\left(\mathbf{S}_{\mathbf{o}} - \mathbf{S}_{\mathbf{ol}}\right) = K_{ro(g)}\left(S_o\right) - K_{ol}\left(S_{ol}\right)$ $\mathbf{S}_{\mathbf{gr}} = \frac{S_g^{\max}}{1 + C_g S_g^{\max}}; \mathbf{C}_{\mathbf{g}} = \frac{1}{S_{grw}} - \frac{1}{S_g^{\max}}; \mathbf{S}_{\mathbf{gf}} = \frac{1}{2}\left[\left(S_g - S_{gr}\right) + \sqrt{\left(S_g - S_{gr}\right)^2 + \frac{4}{C_g}\left(S_g - S_{gr}\right)}\right]$	Blunt's Model

Aziz and Setarri[23] Modification (1979)

Both of Stone's models were modified by Aziz and Setarri [23]. Absolute permeability was used as the basis for calculating relative permeability.

For Stone's first model

$$K_{ro} = \frac{S_o^{*}\left(K_{row} K_{rog}\right)}{\left\{K_{rocw}\left(1 - S_w^{*}\right)\left(1 - S_g^{*}\right)\right\}}$$

For Stone's second model

$$K_{ro} = \left(K_{row} + \omega K_{row}\right)\left(K_{rog} + \omega K_{rgo}\right) - \omega\left(K_{rwo} + K_{rgo}\right)$$

$$\omega = \frac{K_{rocw}}{K}$$

Since there is no physical basis for the modification, it is not necessary to limit ω to this value. Decreasing the value of ω from 1.0 results in the prediction of greater oil permeability in the three – phase region with no effect on two – phase oil permeability prediction.

Fayers and Mathews [24] Modification (1979)

Based on comparison made between Stone's models as normalized by Aziz and Setarri and the published experimental data from Corey *et al* [6] , Saraf and Fatt [16], and Holmgren and Morse [25], Fayers and Matthews [24] suggested a modification of Stone's first model for S_{om} values :

$$S_{om} = \omega S_{orw} + (1-\omega) S_{org}$$

$$\omega = 1 - \left[\frac{S_g}{\left(1 - S_{wc} - S_{org}\right)}\right]$$

A second modification was developed in the presence of trapped gas:

$$S_{om} = S_{orw} - 0.5 S_g$$

They concluded that the normalized first model with the recommended form for S_{om} is superior to normalized Stone's second model by Aziz and Setarri.

Fayers's [26] Modification (1987)

Fayers [26] suggested an additional form for S_{om} to allow a better fit of the low permeability isoperms.

$$S_{om} = \omega S_{orw} + (1-\omega) S_{org}$$

$$\omega = 1 - \left[\frac{S_g}{\left(1 - S_{wc} - S_{org}\right)}\right]$$

The parameter ω offers a free choice when adequate experimental data are available for fitting S_{om}.

Aleman's Modification (1986)

Baker [15] reported Aleman's modifications of Stone's first model. Aleman has suggested another approximation for S_{om} using free parameters (ω and β) to fit the curvature of the zero – oil permeability isoperm.

$$S_{om} = S_{orw}\left[\frac{\left(S_w - S_{wc}\right)}{\left(1 - S_{wc} - S_{orw}\right)}\right]^{\omega} + S_{org}\left[\frac{S_g}{\left(1 - S_{wc} - S_{org}\right)}\right]^{\beta}$$

Parker *et al* [27] Model (1987):

The model of Parker *et al* [27] was based on a model for two – phase relative permeability developed by Mualem [27]. The permeability to a particular phase in Mualem's [27] model is assumed to be proportional to the square of the mean hydraulic radius of the pores occupied by that phase; this contradict with the model of Corey's *et al* [10] in which the phase permeability was assumed proportional to the mean hydraulic area or the mean square hydraulic radius. The model indicates in general, that oil permeability is a function of both water and oil saturation.

$$K_{ro} = \left(\bar{S}_{o}\right)^{0.5}\left[\left\{1 - \left(\bar{S}_w\right)^{\frac{1}{\lambda}}\right\}^{\lambda} - \left\{1 - \left(\bar{S}_l\right)^{\frac{1}{\lambda}}\right\}^{\lambda}\right]^2$$

$$\lambda = 1 - {}^{1}\!/_{n'}$$

The fitting parameters n' and hence λ can be obtained by curve fitting of saturation capillary pressure data or by fitting two – phase relative permeability data. Use of the model is therefore limited to cases where the parameter provides a satisfactory fit to the two phase data.

Aleman's [28] Model (1984):

This model was based on statistical structural model. It is assumed that the saturation changes in the direction of decreasing intermediate wetting phase saturation. Aleman's *et al* model [28] can be written as:

$$K_{ro} = K_{ro}\left(I\right)^{-\Delta}$$

Where K_{ro} (I) is the relative permeability to oil predicted by Stone's first model and Δ is a correction term. This model is sensitive to the value of S_{om}, and may predict incorrect oil permeability for values of S_{om} which are too small. The model should be used only if examination of the predicted isoperms shows reasonable behavior.

Baker's Model (1989):

Delshad and Pope [29] presented this model which was proposed by Lee Baker. The model is based on an interpolation between two phase data. The two – phase relative permeability data can be obtained from experimental data or can be estimated using two-phase models. The proposed expression is:

$$K_{ro} = \frac{\left[K_{row}\left(S_w - S_{wr}\right) + K_{rog}\left(S_g - S_{gr}\right)\right]}{\left[\left(S_w - S_{wr}\right) + \left(S_g - S_{gr}\right)\right]}$$

Pope's [29] Model (1989):

Pope's model was presented by Delshed and Pope [29].The model is independent of the two -phase data and is given in the following form:

$$K_{ro} = K_{rocw}\begin{bmatrix}\alpha\left(\bar{S}_o\right)^{\beta}\left(1-\bar{S}_w\right)^{\eta} + \\ \left(1-\alpha\right)\left(\bar{S}_o\right)^{\sigma}\left(1-\bar{S}_g\right)^{\delta}\end{bmatrix}$$

The parameters in the equation can be calculated using the following two methods.

Method A: The parameters were defined such that the model at the two phase data limits takes the value of experimental two phase data.

Method B: parameters β and δ were fixed by history matching the two phase oil /gas and oil/water relative permeability data, while other parameters were chosen by history matching the three-phase data.

Lake's [29] Model (1989)

Delshad and Pope[29] presented a model by Lake to predict the micro emulsion relative permeability during three-phase oil – water micro emulsion flow. Delshed and Pope[29] reported that Mojdeh[29] *et al* applied the same concept to estimate the oil relative permeability during the flow of gas, water and oil.

$$K_{ro} = K^{\circ}_{ro}\left[\frac{\left(S_o - S'_{or}\right)}{\left(1 - S_{wr} - S_{gr} - S'_{or}\right)}\right]^{e\circ}$$

The parameters K^0_{ro} and e^0 are as defined in Table 2.

Corey-type Model (1989)

Delshed and Pope [29] presented a model by Mojdeh [29] *et al*. They assumed that the dependence of oil relative permeability on two saturations was through the dependence of residual oil saturation on two saturations. The following model was presented:

$$K_{ro} = K^{\circ}_{ro}\left[\frac{\left(S_o - S'_{or}\right)}{\left(1 - S_{wr} - S_{gr} - S'_{or}\right)}\right]^{eo}$$

$$S'_{or} = F\left(S_g, S_w\right)$$

Marek's [20] Model (1991):

Marek *et al* [20] proposed a model that includes "trapped gas effect" which leads to a reduced water flood residual oil saturation.

$$K_{ro} = K_{row}\,S_{oew}\left(\frac{S_{wf}}{S_{wf} + S_g}\right) + K_{rog}\,S_{oeg}\left(\frac{S_g}{S_{wf} + S_g}\right)$$

The parameters S_{oew} and S_{oeg} are as defined in Table 2.

The model was compared with Stone's second model using measured three-phase data for mixed wettability of carbonate sample having bimodal pore size distributions. The proposed model appears to be superior to Stone's second model especially in the low oil saturation range.

Blunt's Model [1] (1999)

Blunt [1] presented an empirical model for three phase relative permeability that overcomes the limitations of current formulations, such as Stone's models. It provides a self-consistent treatment of wettability, changes in hydrocarbon composition, different saturation paths, and the trapping of oil, water and gas .The theoretical development is motivated by a review of recent three phase experiments. The model is based on saturation–weighted interpolation between the two-phase values. To account for the effects of wettability, a saturation-weighting all three phases can be applied by writing the relative permeability as a unique function of a flowing saturation, which ensures smooth changes in relative permeability .The model is presented in the following form:

$$K_{ro} = \alpha' \frac{\left[\left(S_w - S_{wi}\right)\left\{a_\circ K_{row}\left(S_{ofb}\right) + b_\circ K_{rgw}\left(S_{ofb}\right)\right\}\right]}{\left(S_w - S_{wi}\right) + \left(S_g - S_{gr}\right)}$$

$$+\alpha' \frac{\left[\left(S_g - S_{gr}\right)\left\{a_\circ K_{ob}\left(S_{ofb}\right) + b_\circ K_{rgw}\left(S_{ofb}\right)\right\}\right]}{\left(S_w - S_{wi}\right) + \left(S_g - S_{gr}\right)}$$

$$\alpha' K_{ol} S_{ol} + \beta' \left[\frac{S_o}{\left(S_o + S_g\right)}\right]\left[a_\circ K_{row} S_{hf} + b_\circ K_{rgw} S_{hf}\right]$$

The model was tested against the data of Oak *et al* [17] and Oak [19]. It showed that it was necessary to include layer drainage and oil trapping to predict three-phase oil relative permeability at low oil saturation accurately.

Moulu's [2] Model (1999):

Three-phase relative permeability is obtained by history matching gas injection experiments performed in water–wet and oil–wet porous media. This model is properly modified to take into account different wetting conditions through a wettability index. It is shown that the model is able to correctly predict oil recovery and breakthrough time for a series of gas injection tests performed under secondary and tertiary conditions.

In this model, the porous medium is described by the capillary curve of mercury intrusion: The three fluids are assumed to flow together in concentric layers within the same fractal pore: the wetting phase along the rock walls, the gas phase in the center of the pore, the third phase sandwiched between the two others.

In water wet condition[3], oil relative permeability can be calculated using the following Equation:

$$K_{ro} = K_{ro}\left(2Ph\right)\left[\left(S_l\right)^{\left(\frac{4-DL}{2-DL}\right)} - \left(S_w + S_{or}\right)^{\left(\frac{4-DL}{2-DL}\right)}\right]$$

In this equation, the irreducible water saturation S_{wi} is assumed to be immobile. S_{or} is a part of the residual oil S_{orw} that corresponds to a given water saturation S_w, where S_{orw} is the maximum residual oil saturation left in place by a water flooding. K_{ro} $(2ph)$ is the value of the oil relative permeability determined for a water/oil imbibitions test. When only water and oil are present. D_L is the linear fractal dimension (The value of which corresponds to the surface roughness).

Moulu *et al*[2] presented a general expression for calculation of oil relative permeability for water-wet and oil-wet systems as follows:

$$K_{ro} = \left(1-m\right)\left(S_o^{\phi} - S_{oi}^{\phi}\right) + mK_{ro}\left(Ph\right)\left(S_l^{\phi} - \left(S_w + S_{or}\right)^{\phi}\right)$$

Where S_{oi} is defined as the irreducible oil saturation, $\phi = (4\text{-}DL)/(2\text{-}DL)$ and the exponent m represents the general wettability index which can be calculated using the following equation:

$$m = \frac{WI + 1}{2}$$

Where:
m = 1 water-wet
m = 0 oil-wet

COMPARISON STUDIES BETWEEN THE MODELS

Baker [15] made a comparative study between the models of Stone[4,5], Hirasaki [21], Corey *et al* [10], Naar and Wygal [11], Land [14], Aleman [28], and Parker *et al* [27] and additional models (saturation weighted interpolation and true line interpolation).The comparison indicates that each model is capable of representing three-phase oil permeability in the high oil relative permeability region but, in the low permeability region, the differences in the models were much more evident. In general, the models were often not very good predictors of the experimental data. In most cases, a straight line interpolation or saturation-weighted interpolation between the permeability at the two-phase boundaries of the three-phase flow region provided a better fit of the experimental data than did the theoretically based models. Delshed and Pope[29] made a comparison between the models of Stone[4,5], Pope[29], Baker[29], Lake[29], Parker[27] and Corey-Type[29]. They used the experimental data of Corey *et al*[10], Donaldson and Dean[7], and Saraf[8] *et al*. The predicted oil relative permeability was far from the experimental data of Donaldson and Dean[7]. That may have been partially due to the assumptions made in all models that water and gas relative permeability was function of its own saturation. This was done in spite of the fact that Donaldson and Dean[7] concluded that gas and water relative permeability depends on the saturation distribution of all phases. The comparative study indicated that agreement between the computed oil isoperms from the models of Baker[29], Pope[29], and Parker[27], and experimental data of Corey *et al*[10] was reasonably good. The Corey-Type[29] model gave good predictions for high and low isoperms while Stone's[4,5] models performed better for mid range relative permeability values. Oak[19] in his extensive study of eight cases of saturation history, concluded that experimental data does not support the assumption that two-phase relative permeability data can be used to predict three-phase relative permeability data with a similar saturation history. According to this conclusion, all the models that are based in the above assumption such as Stone's models can not predict three-phase oil relative permeability accurately. It may be possible to conclude that the commonly used models such as Stone's[4,5], often do not give accurate predictions of the experimental data and clearly indicate the need for better three-phase relative permeability models. Recently, the authors have made a comparison study between the models of Moulu[2,3], Blunt[1] and the Stone[4,5] using the data of Corey *et al*[10], Donaldson and Dean[7], and Saraf[8]. The results are shown in Figures 1, 2 and 3 which indicate that the new models were much better than the commonly used models of Stone[4,5] but failed to fit the experimental data.

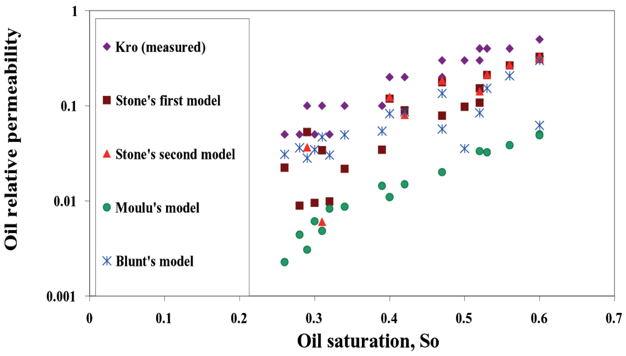

Fig. 1. Comparison with Corey's data.

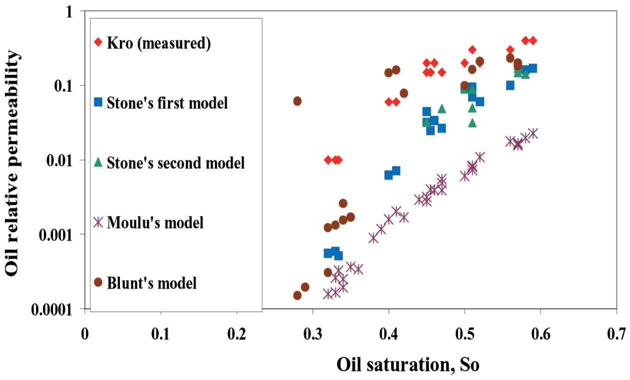

Fig. 2. Comparison with Donaldson and Dean's data.

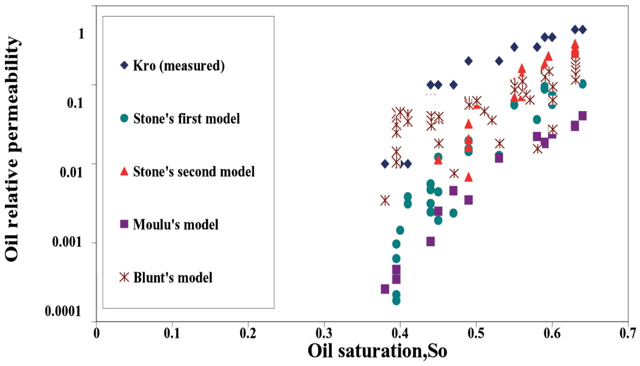

Fig. 3. Comparison with Saraf's data.

SUMMARY AND CONCLUSIONS

On the basis of the work presented here on the theoretical model of three-phase oil relative permeability, the following points are concluded:

1. Most of the theoretical models failed to predict three-phase oil relative permeability close to the experimental data especially in the low oil relative permeability region. Note that "theoretical" models

are really empirical models since there is little genuine justification for them in terms of the fundamental physics of three-phase flow in porous media.

2. Stone's models and their modifications, which are widely used, have failed to predict three-phase oil relative permeability close to experimental data at low oil saturation and an optimized formula for estimating the value for the minimum residual oil saturation (S_{om}) has not been found. The new models of Moulu [2,3] and Blunt [1] show better agreement with experimental data but the errors are still high.

3. No general model has yet been found and more work is required in this area with more attention to the experimental data in order to verify the theoretical models.

NOMENCLATURE

a = Defined in Table 2
a_o = Degree of oil-ness defined in Table 2
b_o = Degree of gas-ness defined in Table 2
C = Trapping characteristic defined in Table 2
C_g = Land trapping constant for gas.
C_h = Land trapping constant for hydrocarbon.
C_o = Land trapping constant for oil.
C_{so} = oil spreading coefficient, F/L, N/m.
D_L = linear fractal dimension.
e_o, e_g, e_w = Exponent of relative permeability curve of oil, gas and water defined in Table 2
G = Interpolating function defined in Table 2
K = Permeability.
K_{ob} = Bulk oil relative permeability defined in oil relative permeability from layers.
K_{ol} = Oil Relative permeability from layers.
K_r = Relative permeability.
$K^o{}_{rg}$ = Endpoint relative permeability of gas defined in Table2
K_{rgo} = Two-phase gas relative permeability flowing with oil. Defined in Table 2
S_h = Hydrocarbon saturation.
S_{hf} = Flowing hydrocarbon saturation defined in Table 2
S_{hr} = Residual hydrocarbon saturation defined in Table 2.
S_l = Liquid saturation.
K_{rgw} = Two-phase gas relative permeability for gas injection into water.
K_{ro} = Three-phase oil relative permeability.
$K^o{}_{ro}$ = Endpoint relative permeability of oil defined in Table 2.
K_{rocw} = Oil relative permeability in the presence of connate water.
K_{rog} = Two-phase oil relative permeability flowing with gas defined in Table 2.
K_{row} = Two-phase oil relative permeability flowing with water defined in Table 2.
$K^o{}_{rw}$ = Endpoint relative permeability of water defined in Table 2.
K_{row} = Two-phase water relative permeability flowing with oil. Defined in Table 2.
m = Wettability index defined in Table 2.
Pc = capillary pressure.
$S^*{}_{fw}$ = Defined in Table 2.
S_g = Gas saturation.
$\overline{S}_g$ = Defined in Table 2.
S_{gf} = Flowing gas saturation.
$S'{}_{gi}$ = Initial gas saturation.
S_{gr} = Residual saturation of gas.
$S'{}_{gr}$ = Residual gas saturation.
S_{wc} = Connate-water saturation.
S_{wf} = Flowing water saturation.
S_{wi} = Initial water saturation.
S_{wr} = Residual saturation of water.
S_{lr} = Residual liquid saturation.
S_o = Oil saturation.
$S^*{}_o$, $\overline{S}_o$, S_{ob} = Defined in Table 2.
S_{oeg} = Effective three-phase oil saturation (gas-oil system).
S_{oew} = Effective three-phase oil saturation (water-oil system).
S_{of} = flowing oil saturation defined in Table 2.
$S'{}_{of}$ = free oil saturation.
$S^*{}_{of}$ = Defined in Table 2.
S_{ofb} = flowing bulk oil saturation.
S_{oi} = initial water saturation.
S_{on} = Defined in Table 2.
S_{or} = residual oil saturation.
$S'{}_{or}$ = residual oil saturation Defined in Table 2.
S_{org} = residual oil saturation to flowing gas.
S_{orw} = residual oil saturation to flowing water.
S_w = water saturation.
$\overline{S}_w$, $S'{}_w$ = Defined in Table 2.
$\&w$ = water multiplier.
$\&g$ = gas multiplier.
$\omega\Delta$, λ = Defined in Table 2.
α = free parameter.
β = free parameter.
η = free parameter.
σ = free parameter.
δ = free parameter.
α', β' = Defined in Table 2.
γ = Interfacial tension, F/L, N/m.
ρ = density, M/L^3, Kg/m^3.
$\Delta\rho_o$ = reference density difference,M/L^3,K/m^3.
S_{om} = minimum residual oil saturation.
ψ = Defined in Table2.

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