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The Role of Pore Connection in Hydraulic Conductance of Sedimentary Rocks

G. Szigeti*

دور الربط المسامي في التوصيل الهيدروليكي في الصخور الرسوبية

ج. سيفاتي

في هذه الورقة تمت دراسة البنية الداخلية للصخور الرسوبية كمظومة من المسام الموصولة، حيث تم افتراض مكعب لشبكة منتظمة من المسام والقنوات الموصلة بينها بحيث تكون في حالة عشوائية من الفتح أو الإغلاق. ودرست إمكانية وجود أو عدم وجود توصيل هيدروليكي بين الوجهين المتقابلين لمكعب عينة الصخر، اعتماداً على نسبة عدد القنوات المفتوحة إلى عدد القنوات عامة.

وتم إيجاد الحل التكراري للشبكة العشوائية الموصوفة، وكذلك تشبيهه في بعدين على الحاسوب، كما وتم تشبيه نموذج مبسط منه في ثلاثة أبعاد. وقد أعطى تحليل توزيع الاحتمالات صورة كمية حية وواضحة عن دور الربط المسامي في الصخور في الحركة الهيدروليكية. وكنتيجة عامة تخص الصخور ذات البنية التكعيبية المشار إليها أعلاه يمكننا وصف دور الربط المسامي في التوصيل الهيدروليكي بالآتي: إذا كانت نسبة 6/1 من عدد القنوات مفتوحة، فإن الصخر يعتبر غير نفاذي؛ أما إذا كانت النسبة 2/1 من مجموع القنوات مفتوحة، فإن الصخر يعتبر حينذاك نفاذي.

Abstract: *The inner structure of sedimentary rocks is studied as a system of interconnected pores. A regular cubic network is considered for the pores and the channels connecting them with the latter being randomly open or closed. The problem of whether or not there is a hydraulic connection between two parallel faces of a cubic rock sample, depending upon the ratio of open edges to all the possible edges, is studied.*

Recursive solution for the described random network problem is given. The recursive solution in two dimensions is simulated on the computer as the simplest three-dimensional case.

Analysis of the probability of distributions gives a vivid, quantitative picture about the role of pore connection in hydraulic movement. As a rule of thumb for rocks with the above cubic structure, this role can be described as follows: if only $^1/_6$ of the channels are open the rocks are not permeable but if $^1/_2$ of the channels are open the rocks are permeable.

INTRODUCTION

A porous medium only allows the displacement of fluids to the extent that its pores are interconnected. (Perrodon, 1983).

*Petroleum Research Centre, P.O. Box 6431, Tripoli, Libya

The percentage of the channels which should be open (or closed) to allow the rocks permeable or impermeable is dealt with in this study.

Let us consider a certain sedimentary rock with a texture and structure which represent the intergranular porosity in separate pores, while they are connected with thin, narrow tubes or channels. The tubes or channels can be open or closed against pore to pore hydraulic movement.

This model seems to be applicable to mature, consolidated sandstone rather than to unconsolidated sandstone of higher porosity, where the porespace does not necessarily allow its classification as pore and channel. In unconsolidated sandstone plane-like connections can represent a large part of non-solid volume. For real rocks, the question about the open or closed character of the channel is regulated by the presence and development of cementing material, or by the type and development of dispersed shale.

The determination of the numerical value of the permeability of a rock as a function, among others, of the permeability of each channel (zero for the closed one and a given constant for the open ones) has a highly complex character. Consequently, the model presented in this paper is restricted to determination of the permeable or impermeable character of the rock. In spite of this restriction, the model gives a vivid picture about the fluid movement in porous media. In this respect, the character of the results derived here can be compared with that of the porosity calculation of different regular spherical packings (Amyx, *et al.*,1960). It does not play a direct role in the solution of reservoir engineering tasks but gives a fixed scale for the interpretation of porosity data in the solution of engineering problems. This picture also helps to make a visual, qualitative estimation of permeability of a core sample on the basis of microscopic thin sections. Despite the fact that no special hydraulic properties are taken into account, one can not apply the model for the questions of electrical conductivity, because shale (and cementing materials), in general, are not perfect insulators.

Percolation theory deals with the conductivity and permeability of random networks. The achieved results are summarised by monographs (Dullian, 1992; Stauffer and Aharony, 1992). The main method of the percolation theory, since the earliest result (Broadbent, 1954), is the Monte Carlo simulation of the random network. Consequently the nodes of the network should be relatively high, in the range of 10^3-10^4 nodes (Seliakov and Kadet, 1996).

The present work deals with random network as well, however as it will be explained latter on, there is one essential assumption, namely, only one way connections are considered in the direction of the fluid flow. This new assumption gives the basis for the application of recursive solution, which, as it is proven in this paper, dramatically reduces the number of necessary nodes for getting the stabilised solution. This feature of the solution gives a good chance for the application of the recursive method for further tasks. In general, the results derived here are similar to that of the percolation theory, e.g. the percolation thresholds for the square and cubic arrangements are close to that of Korvin, 1992. The aim of the present work is the introduction of the recursive method, rather than the detailed comparison of the two, similar but not identical, tasks.

SETTING UP THE HYDRAULIC CONNECTION MODEL AND QUANTIFICATION OF THE PROBLEM

Suppose the following construction is of a logical rather than an experimental character consisting of spheres of uniform diameter of one length unit with centres in the integer grid points (i_x,i_y,i_z), where $1 \le i_x \le n_x+1$; $1 \le i_y \le n_y+1$; $1 \le i_z \le n_z+1$. Exerting pressure, as a compaction model of the rock, on each of the side walls of the packing can change the spheres shape into cubes with rounded edges while vertexes have the shape of common dice. The centres are described with the same (i_x, i_y, i_z) grid points but with a reduced length. Shifting the coordinate system of the packed model by the vector $(^1/_2, ^1/_2, ^1/_2)$ and considering (i_x, i_y, i_z) only for $1 \le i_x \le n_x$; $1 \le i_y \le$ ny; $1 \le i_z \le n_z$, gives the pore volume in the form of $n_x n_y n_z$ separate pores. Each inner pore volume is connected to six others by narrow channels. It is also assumed, as a model of further consolidation of the rock, that any of the unit length edges can be randomly blocked against hydraulic movement by cementation or by dispersed shale fraction. Assuming high hydraulic pressure on the $i_z = 1$ plane and low pressure on $i_z = n_z$ plane the question of whether fluid movement is possible between the two parallel faces is posed (Fig. 1).

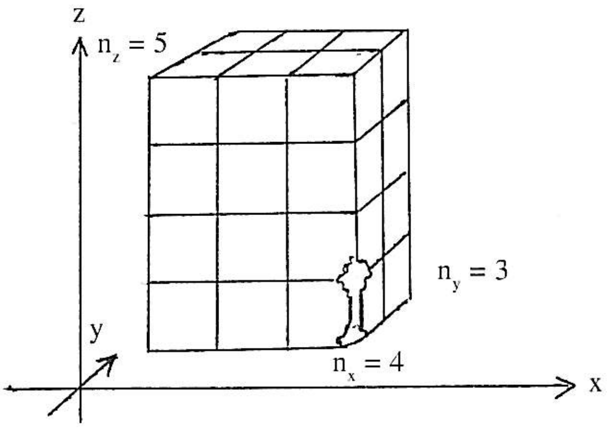


Fig. 1. Cubic arrangement for modelling the grains, the pores and the pore connections.

The network of Fig. 1 has $n_x\, n_y\, n_z$ pore centre points and the number of edges are denoted as $N_{edges}^{block}(n_x, n_y, n_z)$. If we group edges in that of x, y and z direction, then their numbers are arrived at in the form:

$$N_{edges}^{block}(n_x, n_y, n_z) = n_y\, n_z\, (n_x - 1) + n_x\, n_z\, (n_y - 1) + n_x\, n_y\, (n_z - 1) = 3\, n_x\, n_y\, n_z - n_x\, n_y - n_x\, n_z - n_y\, n_z \quad (1)$$

The derivation requires the number of edges on plane surface as well, as an example, on the i_z=1 plane. Introducing the $N_{edges}^{rect}(n_x,n_y)$ notation for the number of edges for the rectangle, similar to eq.(1) but in two dimensions, gives:

$$N_{edges}^{rect}(n_x,n_y) = n_y\, (n_x - 1) + n_x\, (n_y - 1) = 2\, n_x\, n_y - n_x - n_y \quad (2)$$

The question about the presence or absence of connection between the parallel faces depends on the ratio of the sizes (dimensions). To be conductive a thin layer ($n_z << n_x$ and $n_z << n_y$) requires only a few open connections, while an elongated object ($n_z >> n_x$ and $n_z >> n_y$) requires a higher number of open channels.

Thus, a cubic rock sample is examined. The fluid enters into the pores, given by $i_z = 1$, should reach the face given by $i_z = n_z$. The cube corresponds to the situation where the n-width pore openings require the same number of edges for the fluid molecules to travel in order to reach the opposite face. So, the size of our hydraulic cube for z direction conductance is defined as

$$n_x = n_y = n \qquad \text{and} \qquad n_z = n + 1 \quad (3)$$

The hydraulic pressure should only have a z direction on the $i_z = 1$ and $i_z = n_z$ faces, thereby deleting from the system the x and y edges direction from these two parallel planes. For the sake of conciseness, the object remaining after deleting the superfluous edges is referred to as *hydraulic network of n- story* (Fig. 2).

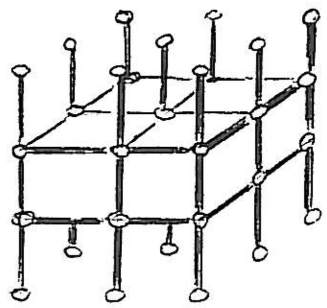


Fig. 2. Hydraulic network of 3-story.(3D, n=3).

The number of edges, denoted by $N_{edges}^{hyd}(n, n, n+1)$, of the hydraulic network can be easily obtained. By virtue of the construction,

$$N_{edges}^{hyd}(n_x, n_y, n_z) = N_{edges}^{block}(n_x, n_y, n_z) - 2\, N_{edges}^{rect}(n_x, n_y) \quad (4)$$

Substituting eq.(3) into eq.(4) and applying eq.(1) and eq.(2) gives:

$$N_{edges}^{hyd}(n, n, n+1) = 3\, n^3 - 4\, n^2 + 2\, n \quad (5)$$

Exact mathematical form of the problem is the following. Having a hydraulic network of n-story and a fixed i integer number $1 \le i \le N_{edges}^{hyd}(n, n, n+1)$ all the different hydraulic networks are studied, where the exact i number of channels are open. The question derived from this is:

$C\,(N_{edges}^{hyd}(n, n, n+1), i)$

[where $C(n,k) = n! / ((n-k)!k!)$] number of models, how many allow fluid movement from the $i_z = 1$ plane to the $i_z = n+1$ plane. In all the z-direction edges, fluid movement only in z-direction is assumed (minus z-direction movement is not allowed). This assumption simplifies the task and is adequate from petroleum engineering point of view.

As an example, the 3-story hydraulic network of Fig. 2 is taken. According to eq. (5) we have 51 edges. When $i = 3$ (3 randomly selected open edges) the number of different models become $C(51,3) = 20825$. Reaching the upper plane from the lower one with just three open edges is possible only if all the three edges are vertical and are on the same string (i_x = constant and i_y = constant for the end points of the open edges). This can be done in 9 different ways. Thus, the probability of the event of having a hydraulic connection between the two faces (the ratio of the favourable models to the number of models) is $9/C(51,3) = 0.00043$ which is very small. It is noted that even for $n=3$ for models with a higher number of open edges $i > 3$, it becomes very difficult to determine the number of conductive models.

RECURSIVE SOLUTION AND COMPUTER ALGORITHM OF THE PROBLEM

The solution of the problem for all open edges i from 1 to $N_{edges}^{hyd}(n,n,n+1)$ is needed. The number of models, for $n = 3$, is 2^{51} ($\approx 10^{15}$), while by eq. (5) for $n = 4$, it is 2^{136} ($\approx 10^{40}$). It is fairly possible to build up an algorithm which determines each model (open edge system) whether or not hydraulic connection is established. However, the number of models is enormously high and increasing rapidly with growing n. Consequently, such a direct algorithm is not applicable, starting from some small n, regardless of the capacity of the applied computer system.

Here, examining the hydraulic connection story by story, a recursive solution is given. For a fixed n-story hydraulic network, the dependency of the number of models, which connect any of the points $n_z = 1$ to a given point system of $n_z = j$, upon the number of open edges up to the level $n_z = j$, is studied.

Before setting forth the solution in detail, some general notes should be mentioned.

(a) Any recursion brings useful results only if it can be solved. The main formula of the present paper is complex enough to make it difficult to obtain a solution by hand calculations, but it allows computer solution even for large n values.

(b) The recursive solution, described here, with changes in the main formula, is valid for the general three-dimensional case ($n_y > 1$). However, its description and its computer solution become more difficult than the two-dimensional case $n_y = 1$. The analysis of the three-dimensional general hydraulic network will be given on the basis of the results obtained for the two-dimensional $n_y = 1$ case.

Thus, returning to the recursive solution, the case $n_y = 1$ is supposed, with $n_x = n$ and $n_z = n+1$; where n is a fixed, unchanged integer. For this case, the two-dimensional network, which corresponds to the hydraulic network of n-story can be seen on Fig. 3 for the case $n = 4$. This object will be referred to as *hydraulic ladder of n-story*, and the number of edges of this object is denoted by $N_{edges}^{lad}(n, n+1)$. Applying the same reasoning as was done for eq. (4),and, also using eq. (2) gives:

$$N_{edges}^{lad}(n, n+1) = N_{edges}^{rect}(n, n+1) - 2(n - 1) = n^2 + (n - 1)^2 \qquad (6)$$

Fixing n, alongside the hydraulic ladder of n-story its parts are also defined as follows. Those (i_x, i_z) points and edges for which, in case of points, $i_z \leq i+1$ and edges, at least for one end point $i_z < i+1$ will be referred to as *i^{th} story of n-story hydraulic ladder*. This object and at the same time the number of its edges will be denoted as $N_n(i)$. For $i = 1$, the 1st story consists of n-vertical edges, and for $i = 2$ it consists of the previous vertical edges plus $(n - 1)$ horizontal edges plus n new vertical edges, etc.. In this way

$$N_n(i + 1) - N_n(i) = n + (n - 1) = 2n - 1 \qquad (7)$$

The last, n^{th} story gives back the whole hydraulic ladder of n-story, i.e.

$$N_n(n) = N_{edges}^{lad}(n, n+1) \quad (8)$$

Fig. 3 demonstrates the 2^{nd} story of the hydraulic ladder of 4-story as well.

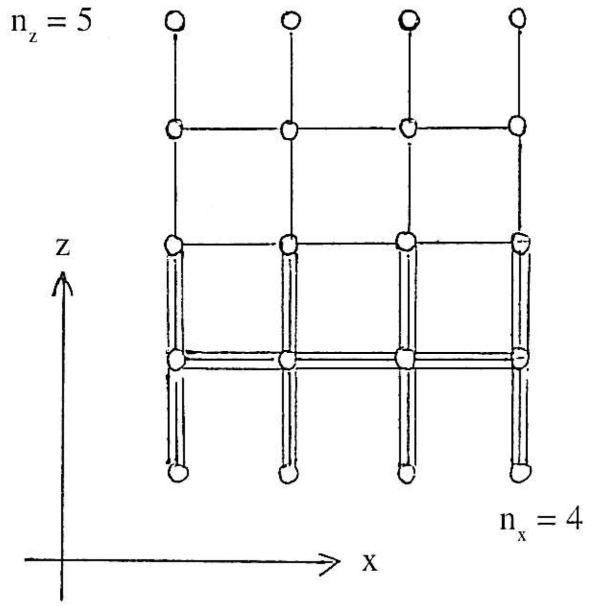


Fig. 3. Hydraulic ladder of 4-story and its 2^{nd} story.(2D, n=4).

Those (i_x, i_z) end points of the i^{th} story of n-story hydraulic ladder for which $i_z = i$ and which are hydraulically connected to the base line ($i_z = 1$) by a given system of open edges are defined as *wet points* (as to the naming to this definition, dry rock sample is considered, which becomes gradually wet as a result of fluid movement through the open connections).

A system of wet points, given by different systems of open edges, can produce any order of zeros and units (0 corresponds to dry and 1 corresponds to wet points). In connection to this fact, $\{Bs(n, j)\}_{j=0}^{2^n-1}$ denotes the set of those binary sequences, which consist exactly n number of 0 -s and 1 -s in the way, that the sequence gives the number j in binary number system [e.g. Bs(5,2) = (0,0,0,1,0)]. Because of the need to refer to the number of units (e.g. number of wet points) in the binary sequence Bs(n,j) it is denoted by |j| (continuing the previous example, |2| = 1). As negative numbers are not used in this paper, there is no danger to read this notation as the absolute value of a number.

For the sake of conciseness, the definition of each integer used in this paper will be fixed. n denotes the size of hydraulic ladder of n-story as before, with a fixed value. m is the m^{th} story of this ladder, $1 \le m \le n$. i is the number of open edges of the ladder up to the m^{th} story, $0 \le i \le Nn(m)$. j and k stand for the binary sequences of wet points of $(m-1)^{th}$ and m^{th} stories, respectively, $0 \le j \le 2^n-1$ or $0 \le |j| \le n$ and the same holds for k. l is the binary sequence which describes the horizontal open edges of the $i_x = m$ row of edges, which are the only horizontal edges, part of the m^{th} ladder but not part of the $(m-1)^{th}$ ladder. 1 stands for the open edges and 0 for the closed ones. Having n-1 horizontal edges, evidently, $0 \le l \le 2^{n-1}-1$ or $0 \le |l| \le n-1$. Fig. 4 illustrates the notations we introduced. Dark points and bold circles represent the wet points of $(m-1)^{th}$ ladder and m^{th} ladder respectively. Single continuous line represents the horizontal open edges of m^{th} ladder (their number is $|l|$), while double lines give these vertical edges of the m^{th} ladder, which are running upwards to wet points.

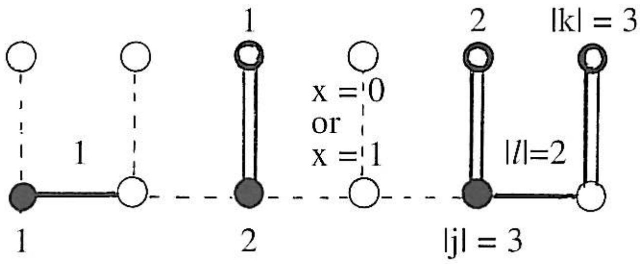


Fig. 4. Notation system for points and edges in different relations to each other.

The question, having the m^{th} story of the hydraulic ladder of n-story and selecting exactly i number of open edges from the m^{th} ladder, can be done in

$$C\,(Nn(m), i) \quad (9)$$

different ways, how many times the end wet stage of the m^{th} ladder is the Bs(n,k) binary sequence. This number is denoted by

$$IN_{Bs(n,k)}\,(N_n(m), i) \quad (10)$$

The relation between the Bs(n,j) wet point system of the $(m-1)^{th}$ ladder and the Bs(n,k) wet point system of the m^{th} ladder is needed for the recursive solution. New wet points can be generated only by vertical edges, consequently all the |k| vertical edges given by Bs(n,k) are open. Because of having only z-direction fluid movement, they establish hydraulic connection only in the case when the $|l|$ new horizontal edges connect all the |k| points of the $(m-1)^{th}$ ladder, which lays directly under the Bs(n,k) end points, to the |j| end wet points of the $(m-1)^{th}$ ladder. This can be realized by quite different Bs(n-1, l). At any given Bs(n-1, l) there can be a certain number of end points of the $(m-1)^{th}$ ladder which remains dry even after connecting the new $|l|$ horizontal open edges. One can open vertical edges to the top of any of these points without changing the wet end point system of the m^{th} ladder. Suppose x vertical edges of this type is placed. This classification of the role of edges is described by the following formula:

$$|N_{Bs(n,k)}(N_n(m), i) = \sum_{j=0}^{2^n-1} \sum_{l=0}^{2^{n-1}-1} \{H_{Bs(n,k)}(Bs(n,j), Bs(n-1,l)) \sum_{x=0}^{Voids(k,j,l)} C(Voids(k,j,l), x) \cdot |N_{Bs(n,j)}(N_n(m-1), i-|k|-|l|-x)\} \tag{11}$$

Here, H is a logical function, which can assume only 0 or 1 values, and describes the hydraulic relation. H is the only one in the case when the Bs(n,j) wet point system of the $(m-1)^{th}$ ladder and the Bs(n-1, l) horizontal open edge system make all the |k| points, which are directly under the |k| points of the Bs(n,k) system; wet. H is zero otherwise.

Voids(k,j, l) are the functions which tell the maximum possible number of x-points, at a given Bs(n,j), Bs(n-1, l) selection, with the Bs(n,k) end point system. If the i-|k|-|l|-x argument of |N is less than zero (which is fairly possible even at H = 1), then |N = 0 should be taken into the right hand side of formula (11) (controversial edge system can not give model realising the Bs(n,k) end stage).

Formula (11) finds any open edge system which produces the Bs(n,k) end wet point system. Indeed, this open edge system is a definite Bs(n,j), Bs(n-1, l) binary sequence. For these two binary sequences H=1, and for the x-value belonging to the open edge system $C \geq 1$ [for the special case x = 0, regardless of the value of Voids(k,j, l), C(Voids(k,j, l), 0) = 1] consequently, the given edge system is taken into consideration in the sum of eq. (11).

Any two edge system, which positively increases the right hand side of formula (11), is given by different edge systems. Indeed, if they belong to different j value, then the two systems of edges differ already in the $(m-1)^{th}$ ladder, if they belong to different l then they differ in the new horizontal edge system. If j and l are the same then they are counted in the sum, i.e. H=1, only in the case if the end stage is Bs(n,k). In this case the two edge system differs at the selection of vertical void edges. This means either x is different for the two systems or the selection from the possible number of horizontal edges (Voids(k,j, l)) is different. This way, the validity of formula (11) is proven.

The recursion character of formula (11) is reflected by the fact that its right hand side consists of the number of models |N only for the $(m-1)^{th}$ story, but for different number of open edges, while the left hand side gives the number of models for the m^{th} story.

To start the recursion, some initial values for the number of models |N establishing the Bs(n,k) end wet point system are needed. Because for the first row ($n_x = 1$) all the points are wet, for the first story (the $n_x \leq 2$ points) Bs(n,k) is regulated exceptionally by the presence or absence of vertical edges, i.e.

$$|N_{Bs(n,k)}(N_n(1), i) = \begin{cases} 1 & \text{if } i = |k| \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Having the initial value from eq. (12) for m=1 and the recursive step given by eq. (11), the number of models |N, which is described at (10), can be calculated, up to the n^{th} story.

This recursive algorithm requires solving by computer the applied logical function H and integer function Voids. Both depend on the new end stage of wet points Bs(n,k), the old one Bs(n,j) and the system of selected horizontal edges Bs(n-1, *l*). There is no major difficulty in these steps. Moreover, their value is not a function of m and therefore there is no need to calculate them only at the first step of the recursion.

The solution of the recursive formula (11) requires a sequence of tables, having 2^n rows for all the different Bs(n,k) values and a changing number of columns. For m = 1, the number of columns is n+1 (we have 0, 1,.. n number of open edges for the first story). For m=2, according to eq. (7), this number increases by 2n-1, the number of possible new open edges, up to m = n, when we have $N_n(n)$ + 1 columns. To fill up the next table we only need the previous one.

	m=1	...	m=n
k |k| Bs(n,k)	$0 \leq i \leq N_n(m)$		
	$N_{Bs(n,k)}(N_n(m),i)$		
	$C(N_n(m),i)$	$2^{N_n(m)}$	

Fig. 5. The structure of Table 1, notations are given by the nomenclature.

The structure of this sequence of tables, with the introduced mathematical notations, is shown in Fig. 5. The three header columns describe the resulting binary sequence of wet points, while for m = 1, m = 2, ... m = n the sub-tables of the sequence of tables are obtained one by one. In each sub-table, the values show the number of different models given by different open edge systems, which give the given Bs(n,k) end wet stage with exactly i number of open edges. The sum of each column is $C(N_n(m), i)$, because $0 \leq k \leq 2n - 1$ describes all the possible end wet stages. These numbers give the last row, while their total is evidently the sum of all the possible different models of edge systems, i.e. $2^{N(m)}$. This number is given as the last number of the last row for each sub-table.

Table 1 reflects the results of the whole process of calculations for n = 3. In the table, Bs(n,k) and the mth story of 3-story hydraulic ladder is given in graphical form. The sub-table belonging to m=1 is given by initial values of eq. (12). The consecutive ones are the result of application of the main recursive formula (11). The last row, which is the sum of the values in the corresponding column, is the $C(N_n(m), i)$ coefficients, and is used to confirm that the program solution of formula (11) is correct. Direct checking of the smaller values, observing some symmetricity in the table and the rigorous inner check of program code are convincing as to the validity of our calculations.

An example of reading data from the table is as follows. The numerical value 209 in the first row of the third sub-table means that selecting 8 open edges out of the 13 potential edges, which can be done C(13,8) = 1287 different ways, 209 models do not establish hydraulic connection between the faces.

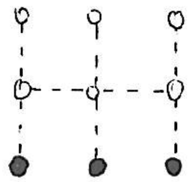
Tables belonging to higher n values can be created similarly. Before reaching the limit of the necessary amount of memory or processing time, the following difficulty is met. The highest integer which can be represented on four bytes is about 2^{32}, while the maximum number of models falling into the same category can be higher than 2^{32}, which means the answer can not be received in exact integer terms. From this n value less accurate real numbers can be applied for getting the classification of models. Working with four bytes integers, the necessity of real numbers comes up at n =5. For n=4 e.g. selecting 13 open edges out of the $4^2 + 3^2 = 25$, which can be done C(25,13) = 5 200 300 by different ways, the number of impermeable models is 2 211 406.

Analysis of the Results

Table 1 gives the probability of having all the end points dry (k = 0) as a function of the probability, that a randomly selected edge is open. First probability is given by the ratio of favourable models to all possible models, i.e. by the ratio of the first row to the last row in the last, m = n, sub-table of Table 1. The second probability is given by eq. (6), by the $i/[n^2+(n-1)^2]$ ratio. The functional relationship between the dependent variable probability of the rock being impermeable and the independent probability of the edge being open is given on (Fig. 6) for n = 2, 3 and 4 values. The probability distributions for different n values are

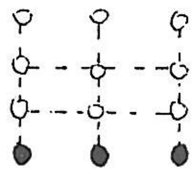
Table 1. Sequences of tables belonging to the 1st, 2nd and 3rd story of hydraulic ladder of 3-story (2D, n=3), describing the distribution of models.

m = 1



m = 2

k	|k|	Bs(n,k)	0	1	2	3		0	1	2	3	4	5	6	7	8	
0	0	○–○–○	1	0	0	0		1	8	25	34	14	2	0	0	0	
1	1	○–○–●	0	1	0	0		0	0	1	7	16	7	1	0	0	
2	1	○–●–○	0	1	0	0		0	0	1	8	17	7	1	0	0	
4	1	●–○–○	0	1	0	0		0	0	1	7	16	7	1	0	0	
3	2	○–●–●	0	0	1	0		0	0	0	0	3	12	6	1	0	
5	2	●–○–●	0	0	1	0		0	0	0	0	1	9	5	1	0	
6	2	●–●–○	0	0	1	0		0	0	0	0	3	12	6	1	0	
7	3	●–●–●	0	0	0	1		0	0	0	0	0	0	8	5	1	
			1	3	3	1	8	1	8	28	56	70	56	28	8	1	256



m = 3

k	|k|	Bs(n,k)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	
0	0	○–○–○	1	13	78	283	677	1078	1089	629	209	38	3	0	0	0	
1	1	○–○–●	0	0	0	1	12	64	183	277	184	64	12	1	0	0	
2	1	○–●–○	0	0	0	1	14	77	211	298	188	64	12	1	0	0	
4	1	●–○–○	0	0	0	1	12	64	183	277	184	64	12	1	0	0	
3	2	○–●–●	0	0	0	0	0	2	23	96	186	138	53	11	1	0	
5	2	●–○–●	0	0	0	0	0	0	4	40	120	103	44	10	1	0	
6	2	●–●–○	0	0	0	0	0	2	23	96	186	138	53	11	1	0	
7	3	●–●–●	0	0	0	0	0	0	0	3	30	106	97	43	10	1	
			1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1	8192

similar. The only two non-trivial probabilities belonging to n = 2, fall very close to the distribution curve belonging to n = 4. This indicates that there is no need to increase n further to estimate higher n probability distributions. The curve tells that if P(edge is open) = 6/25 = .24 then the conditional probability P(rock is impermeable) = .993. If half of the edges are open, then the number of permeable and impermeable models are equal for the examined two-dimensional rock model.

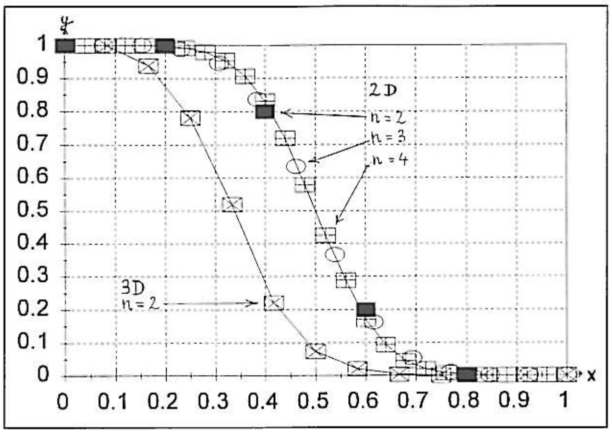


Fig. 6. Probability distribution curves for the hydraulic network in two and three dimensions with different number of story. x axis shows the probability that the edge is open, while y axis gives the probability that the rock is impermeable.

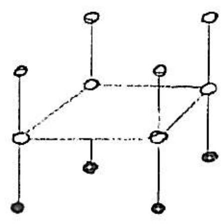
The high stability of the probability distributions of Fig. 6. obtained for the hydraulic ladder of n-story is convincing and that the required distribution from low n values can be obtained. Applying this experience to the 3-dimensional problem of the hydraulic network of n-story, instead of solving by a computer the recursive formula analogue to that of given by formula (11), direct computer program for the 2-story hydraulic network was made. There is no major difficulty in the development of this program, but it does not allow generalisation for any higher n. Indeed, the direct program ran 0.16 seconds on a Pentium processor for n = 2. For n = 3, according to the estimation made using eq. (5), the computer time would be $2^*8/3^*2^{51-12}$ times longer, which is more than 10^4 years.

Results of this 3-dimensional analysis are given in Table 2. According to formula (5), with n = 2, the model has 12 edges. The 16 possible end stages of wet points are indicated in Table 2 by small drawings, without repetition of the symmetrical ones. The very last row of Table 2 gives the required probabilities. The probability distribution belonging to the 3-dimensional hydraulic network is also represented (Fig. 6). Comparing the 3-dimensional distribution with the two-dimensional one, indicates that while for the 2-dimensional case, when half of the edges are open, there is an equal chance for the rock being permeable or impermeable. In the 3-dimensional case one third of the edges open makes these probabilities equal. If only 1/6 part of the edges are open, i.e. P(edge is open) = .166, then P(rock is impermeable) = .939 which means a confidence level of .939 one can declare the rock is

Table 2. Distribution of models for the hydraulic network of 2-story (3D, n=2).

m = 2

k	|k|	Bs(n²,k)	0	1	2	3	4	5	6	7	8	9	10	11	12	
0	0		1	12	62	172	257	176	68	16	2	0	0	0	0	
1	1		0	0	1	12	56	119	87	34	8	1	0	0	0	
2	1		0	0	1	12	56	119	87	34	8	1	0	0	0	
4	1		0	0	1	12	56	119	87	34	8	1	0	0	0	
8	1		0	0	1	12	56	119	87	34	8	1	0	0	0	
5	2		0	0	0	0	1	18	72	64	30	8	1	0	0	
3	2		0	0	0	0	3	26	83	70	31	8	1	0	0	
6	2		0	0	0	0	3	26	83	70	31	8	1	0	0	
9	2		0	0	0	0	3	26	83	70	31	8	1	0	0	
12	2		0	0	0	0	3	26	83	70	31	8	1	0	0	
10	2		0	0	0	0	1	18	72	64	30	8	1	0	0	
7	3		0	0	0	0	0	0	8	58	58	29	8	1	0	
11	3		0	0	0	0	0	0	8	58	58	29	8	1	0	
13	3		0	0	0	0	0	0	8	58	58	29	8	1	0	
14	3		0	0	0	0	0	0	8	58	58	29	8	1	0	
15	4		0	0	0	0	0	0	0	0	45	52	28	8	1	
			1	12	66	220	495	792	924	792	495	220	66	12	1	4096
			1	1	.939	.782	.519	.222	.074	.020	.004	0	0	0	0	



n = 2

impermeable. On the other hand, just with the half of edges open, i.e. P(edge is open) = .5, it can be declared that the rock is permeable with the confidence level 1-.074 = .926. If the ratio of open edges is between 1/6 and 1/2 then judgement can not be made about the permeability character of the rock without having a more detailed knowledge of the inner structure of the cementation or shaliness of the sedimentary rocks.

CONCLUSIONS

1- Complex questions related to the permeability of sedimentary rocks require adequate mathematical models on the basis of the theory of probability.

2- Probabilistic model with a relatively low number of random edges can give a correct answer.

3- Assuming cubic-like arrangement for the pore and pore connection system with randomly selected open edges (connections), and taking a cubic sample from such kind of theoretical rock, analysis of the probability distribution revealed that if only 1/6 of the channels are open the rock is impermeable but if 1/2 of the channels are open the rock is permeable.

NOMENCLATURE

n_x, n_y, n_z number of pores in cubic structure in x, y, z directions, respectively

i_x, i_y, i_z running indices for the pores, integer numbers, e.g. for the x direction, $1 \leq i_x \leq n_x$

$N_{edges}^{block}(n_x, n_y, n_z)$ number of edges (channels) for the 3D block

$N_{edges}^{rect}(n_x, n_y)$ number of edges (channels) for the 2D block (rectangle)

n number of pores in the x and y directions (3D case), or in the x direction (2D case)

$N_{edges}^{hyd}(n, n, n+1)$ number of edges for the hydraulic network of n-story (3D case)

$N_{edges}^{lad}(n, n+1)$ number of edges for the hydraulic ladder of n-story (2D case)

C(n,k) number of combinations, selecting k objects out of n objects, regardless of the order of selection; C(n,k) = n! / ((n-k)!k!)

Bs(n,j) The binary sequence of 0-s and 1-s, which gives the $0 \leq j \leq 2^n - 1$ number in binary number system and which is filled up to n digits from the left by padding zeros if it is necessary. E.g. Bs(5,2) = (0,0,0,1,0).

|j| The number of units (1-s), necessary for describing number j in Bs(n,j). Note that |j| is not a function of n. Continuing the previous item of nomenclature, |2| = 1.

m is the m[th] story of hydraulic ladder of n-story.

i number of open edges in the m[th] story of hydraulic ladder of n story, $0 \leq i \leq N_n(m)$.

j, k describe the system of wet points of the (m-1)[th] and m[th] story, respectively, with the aim of Bs(n,j), Bs(n,k) binary sequences, $0 \leq j \leq 2^n-1$ or, what is the same, $0 \leq |j| \leq n$.

l describes those horizontal open edges, which are part of the m[th], but not part of the (m-1)[th] story of hydraulic ladder of n-story, with the aid of binary sequence Bs(n-1,*l*), $0 \leq l \leq 2^n - 1$ or $0 \leq |l| \leq n-1$.

x number of those open vertical edges, which are part of the m[th], but not part of the (m-1)[th] story of hydraulic ladder of n-story, and, which do not contain wet point as an end point.

$IN_{Bs(n,k)}(N_n(m), i)$ the number of those different open edge systems (network models), which create (give) the Bs(n,k) wet point system for the m[th] story of hydraulic ladder of n story, and which contain exactly i open edges.

$H_{Bs(n,k)}$ (Bs(n,j), Bs(n-1,*l*)) Logical function, assuming only values 0 or 1. H assume 1, if the Bs(n-1,*l*) horizontal open edge system connects the Bs(n,j) wet points to all the points, which are directly under (in -z direction) the Bs(n,k) wet points. H is 0 otherwise.

Voids(k,j,*l*) is the maximum value for x.

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