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ENHANCEMENT OF RESIDUAL TOTAL AEROMAGNETIC ANOMALIES – USING THE HILBERT TRANSFORM

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تحسين شاذات (إشارات) المغناطيسية الجوية باستخدام محول هلبرت

رضا البكباك و ن. ل. موهان

تم استخدام محول هلبرت بغرض تحسين شاذات المجال الكلي للمغناطيسية المتبقية، ولأول مرة، على إشارات من أكثر من 165 خط مسح جوي.

نتطرق في هذه الدراسة إلى العلاقات الرياضية التي تمثل محول هلبرت والمشتقة الأولى، واستنباط المرشح (Filter Generator).

مكنت النتائج المتحصل عليها للإشارات المحسنة من تحديد مصادر شاذات جديدة بالإضافة إلى تأكيد المصادر المحددة سابقاً. كذلك أعطت النتائج مؤشرات على التراكيب الجيولوجية الدفينة بالمنطقة كالفوالق، والتي لم تكن ظاهرة في الخريطة المغناطيسية المتبقية قبل إجراء عمليات التحسين عليها.

ABSTRACT

An attempt is made for the first time to demonstrate the applicability of the Hilbert transform in order to improve the signal resolution from aeromagnetic data over 165 profiles. Filter coefficients (M_j) were generated using the horizontal derivation (A) and its Hilbert transform (H), ($M_j = \Sigma A \times H$) for every profile and subsequently the filter was convolved with the horizontal derivative ($M \times A$) as well as its Hilbert transform ($M \times H$). The validity of the method is established by correlating the filtered aeromagnetic data with drill hole data in locations where anomalous zones are known. The enhanced signals enabled the delineation of new anomalous zones. The resolution of aeromagnetic anomalies, also, gave possible inference to the structural features like contacts and faults in the region.

INTRODUCTION

In continuation of the study of Hilbert transform application for geophysical signal resolution (Mohan *et al.* 1985, 1988; Rama Dass *et al.* 1987) an attempt was made to apply the Hilbert transform for the first time on aeromagnetic data.

The first horizontal derivative (A) and its Hilbert transform (H), were computed from the residual total aeromagnetic data over each profile; and the filter coefficients were cross multiplied with the horizontal derivative ($M \times A$) and its Hilbert transform ($M \times H$) separately. The enhanced signal peaks over a few profiles were correlated with the drill hole data in the area. A new zone was demarcated on the north-northwestern side from the filtered aeromagnetic signal peaks. Also, the signal peaks were correlated with prominent structural features like fault and contacts.

THE HILBERT TRANSFORM

The analysis of aeromagnetic anomalies, in the present case, was based on the Hilbert transform. It was established that the vertical magnetic field and horizontal magnetic field anomalies form the Hilbert transform pair; and equally true with the horizontal derivative and vertical derivative of any component of the magnetic field (Nabighian, 1972; Shuey, 1972; Cerveney and Zahradrik, 1975; and Mohan *et al.*, 1982).

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The Hilbert transform pair are defined by the relation (Erdelyi *et al.*, 1954),

$$A(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{H(\mathscr{G})}{\mathscr{G}-x}\,d\mathscr{G} \tag{1}$$

and

$$H(\mathscr{G})=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{A(x)}{x-\mathscr{G}}\,dx \tag{2}$$

with the principle value to be taken at the singularity of the integrand. Also in the frequency domain, the Hilbert transform of any real function $f(x)$ is defined as

$$H(x) = \frac{1}{\pi}\int_{0}^{\infty}\{\mathrm{Im}\,F(\omega)\cos\omega x-\mathrm{Re}\,F(\omega)\sin\omega x\}\,d\omega \tag{3}$$

where $\mathrm{Re}\,F(\omega)$ and $\mathrm{Im}\,F(\omega)$ are real and imaginary components of the Fourier transform,

$$F(\omega)=\int_{-\infty}^{\infty}f(x)\,e^{-i\omega x}\,dx \tag{4}$$

and ω is the spatial frequency expressed in radians per unit length (Thomas, 1969). The computational procedure was well explained by Nabighian (1972), Shuey (1972) and Mohan *et al.* (1982). Here the Hilbert transform computations were carried out using the Shuey's (1972) FORTRAN IV program, based on convolution algorithm.

GENERATION OF FILTER

Filter generation was based on the following concept (Thomas, 1969). If $f(x)$ is a random process, then the autocorrelation function $R_{ff}(x, x+m)$ wax defined as

$$R_{ff}(x, x+m)=E(f(x).f(x+m)) \tag{5}$$

where $E(.)$ is the expectation operator.

Similarly, for two random processes, $f(x)$ and its Hilbert transform $H(x)$, the cross correlation function $R_{fH}(x, x+m)$ was defined as,

$$R_{fH}(x, x+m)=E(f(x).H(x+m)) \tag{6}$$

if the process $f(x)$ at least wide sense stationary, then eqn. (5) becomes

$$R_{ff}(x, x+m)=R_{ff}(m) \tag{7}$$

a function of the station interval, m, only. Also if $f(x)$ and its Hilbert transform, $H(x)$, are jointly at least wide sense stationary, then eqn. (6) becomes

$$R_{fH}(x, x+m)=R_{fH}(m) \tag{8}$$

By taking the cross correlation of $f(x)$ and its Hilbert transform $H(x)$ (eqn. (8)) with zero lag(i.e. $m=0$), an inner product, M_j, which is a measure of the product of first horizontal derivative, (A), and its Hilbert transform, (H) was defined as (Shimshoni and Smith, 1964; Mohan *et al.*, 1985, 1987, 1988),

$$M_j=\sum_{i=-n}^{n}A_{i+j}\times H_{i+j} \tag{9}$$

The inner product, M_j, gives a measure of the total signal power. The computation of the filter coefficients and subsequent cross multiplication with the horizontal derivative of the residual total aeromagnetic anomaly and its Hilbert transform was done as follows. Filter, M_j, is the summation of the cross multiplied values of the horizontal derivative of the anomaly and its corresponding Hilbert transform, three values at a time (3 point data window), from one end of the profile (eqn. (9)). The process was repeated by sliding the data window along the entire aeromagnetic profile. Each filter coefficient was attributed at the middle point of the data window.

Here the cross multiplication of $M_j\times A_j$ and $M_j\times H_j$ was defined as,

$$MA_1=\sum_{i=n-1}^{n+1}M_i\times A_{i+2} \tag{10}$$

and

$$MH_1=\sum_{i=n-1}^{n+1}M_i\times H_{i+2} \tag{11}$$

where

$1=1, 2, 3, \ldots\ldots N-1$

$n=1, 2, 3, \ldots\ldots N-1$

and $N=$ total number of values

The same procedure was adopted for convolving the filter coefficients (M_i) with the horizontal derivative (A_{i+2}) and its Hilbert transform (H_{i+2}) as was in the case of filter generation. However, the integration interval (or window length) either for generation of filter coefficients or for cross multiplication of the filter A and H was based on purely trial and error.

DATA AQUISITION AND PROCESSING

The aeromagnetic surveys were conducted along the northsouth profiles at a distance of 0.5 km intervals; and tie-lines were oriented in E–W direction; spaced at 3 km intervals. The flight elevation was maintained at 122 ± 24 meters from the mean ground level. Profiles L-111 to L-275 (27°15'-27°45'; 13°15'–14°05'; Fig. 1) were considered for the present study. It may be seen from Fig. 1, towards northern side as well as western side, the significant magnetic anomalous features are absent unlike near the eastern part where anomalies are not only significant but also a lengthy patch, which is distributed up to the end of southern side.

In order to improve the signal resolution the first horizontal derivative of the residual total magnetic data of every profile was computed as

$$T'(x) = (T(x_{n+1}) - T(x_n))/x \tag{12}$$

where n represents the station numbers and x is the station interval in meters. Using eqn. (2), the Hilbert transform of the horizontal derivative was computed for every aeromagnetic profile. The filter, M_j, was computed from the summation of cross-multiplied values of the horizontal derivative of the residual total magnetic anomaly and its corresponding Hilbert transform three values each at a time (3 point data window), from one end of the profile, using equation (9). Each filter coefficient was attributed at the middle point of the data window. From equations (10) and (11), the filter, M_j, was convolved with the horizontal derivative ($M \times A$) and with its Hilbert transform ($M \times H$) and shown in Figures 2 and 3. A significant aspect of filter generation was "a self manipulation" of the observed data itself. Also, it may be noted here that neither the first horizontal derivative nor its Hilbert transform were smoothened prior to the filter generation. Evidently from Figures 2 and 3 the signal resolution was remarkable due to the self generated filter M_j, in all the profiles. This shows that even the noise generated by the first horizontal derivative and its Hilbert transform apart from the inherent noise, was substantially minimised.

DISCUSSION AND CONCLUSION

The signal enhancement procedure outlined above was applied on the 165 aeromagnetic profiles (L-111–L-275) (Fig. (1)). The validity of the method was established by correlating the enhanced signals of the residual total aeromagnetic anomaly with the presence of the anomalous zone in the south eastern side.

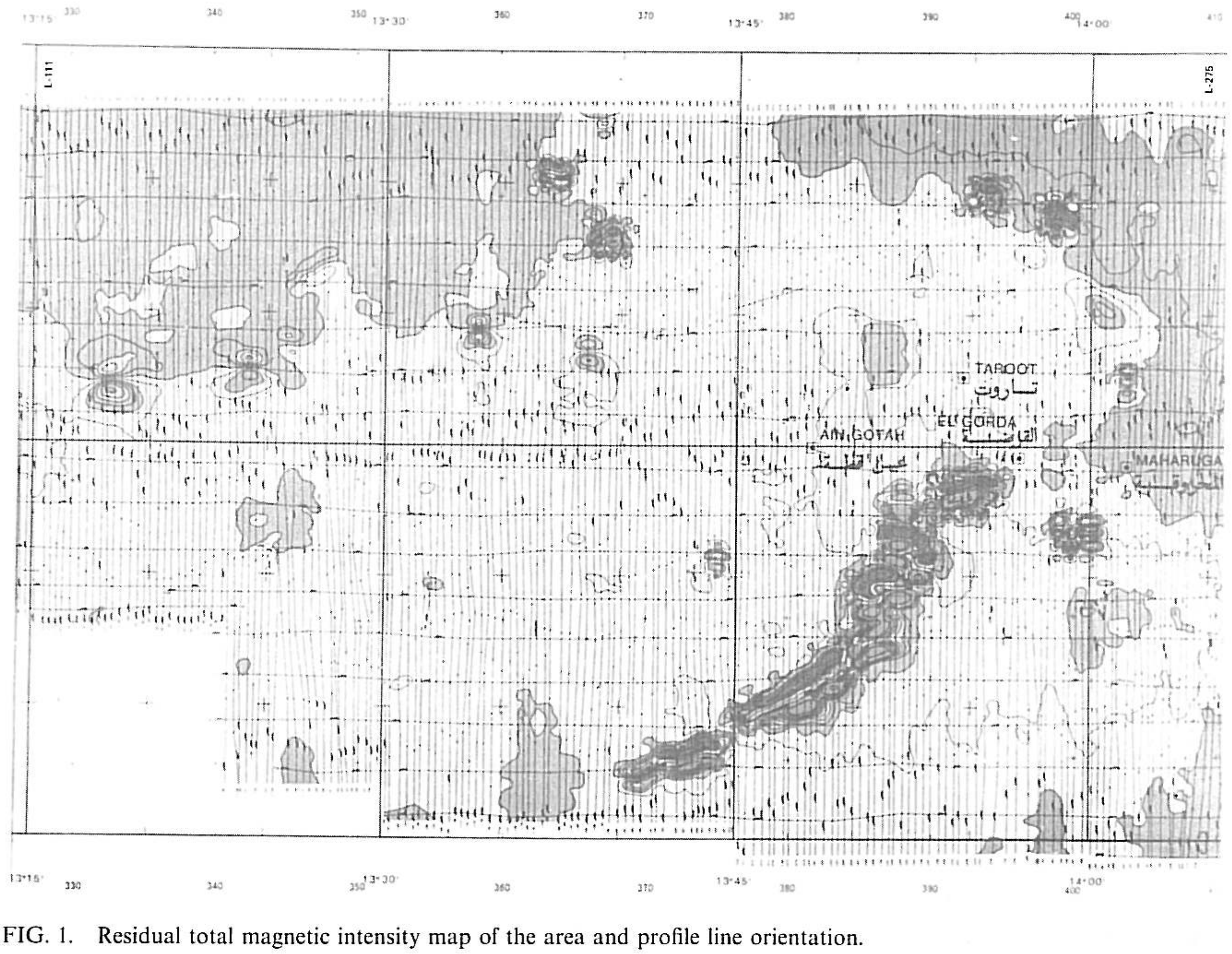

FIG. 1. Residual total magnetic intensity map of the area and profile line orientation.

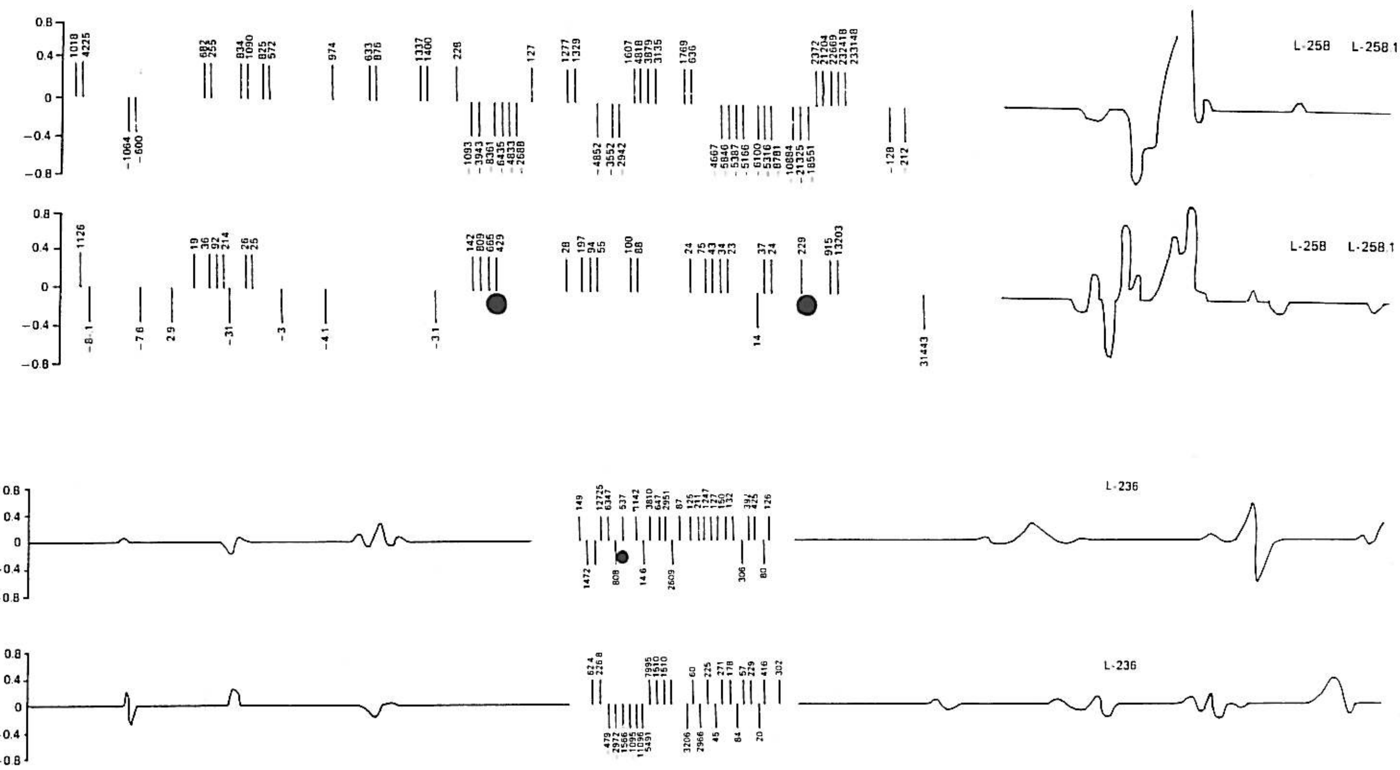

FIG. 2. Convolved horizontal derivative with filter ($M \times A$) and convolved Hilbert transform with filter ($M \times H$) of lines L-236 and L-258 over drill hole locations (solid dot) (L-258.1 is continuation of L-258 with short gap where no data is available).

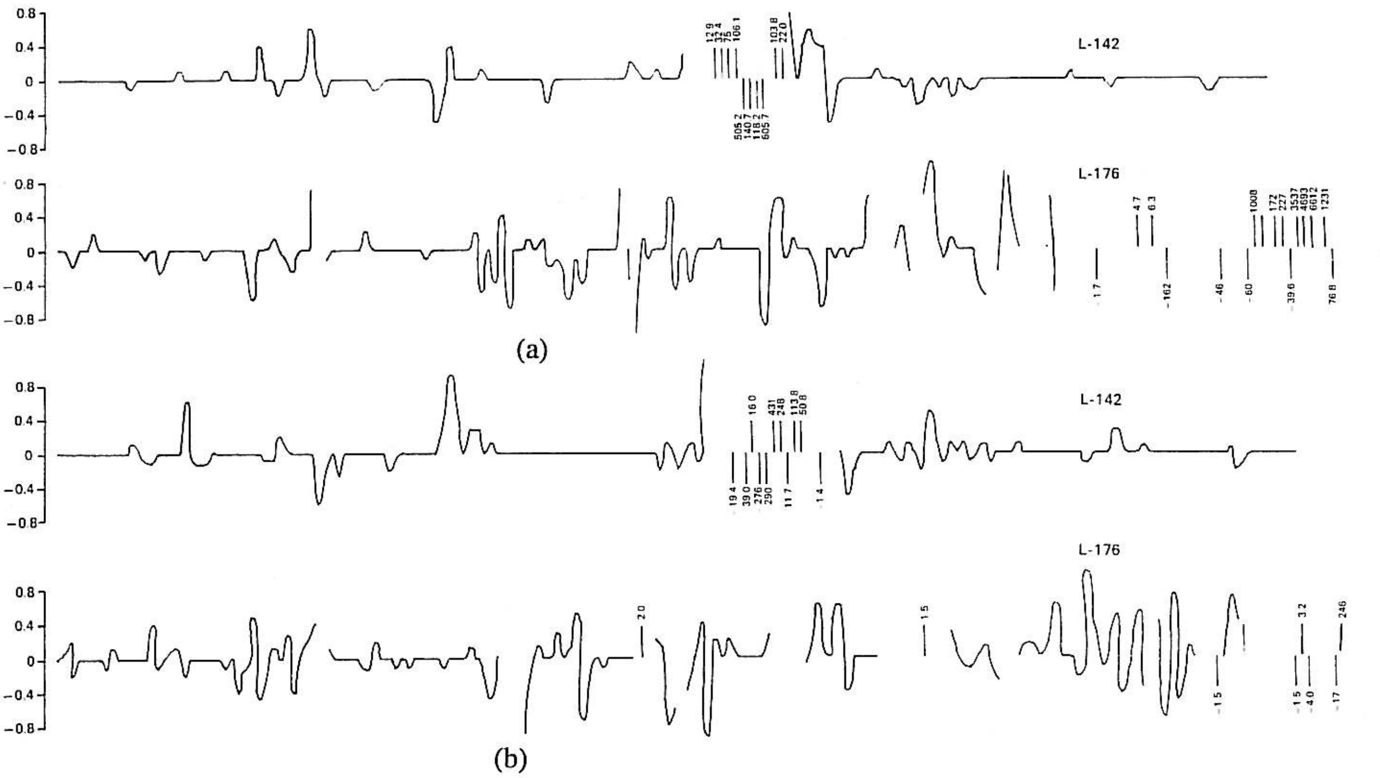

FIG. 3. (a) Convolved horizontal derivative with filter ($M \times A$) of L-142 and L-176 (profile lines through newly delineated zone), (b) Convolved Hilbert transform with filter ($M \times H$) of L-142 and L-176 (profile lines through newly delineated zone).

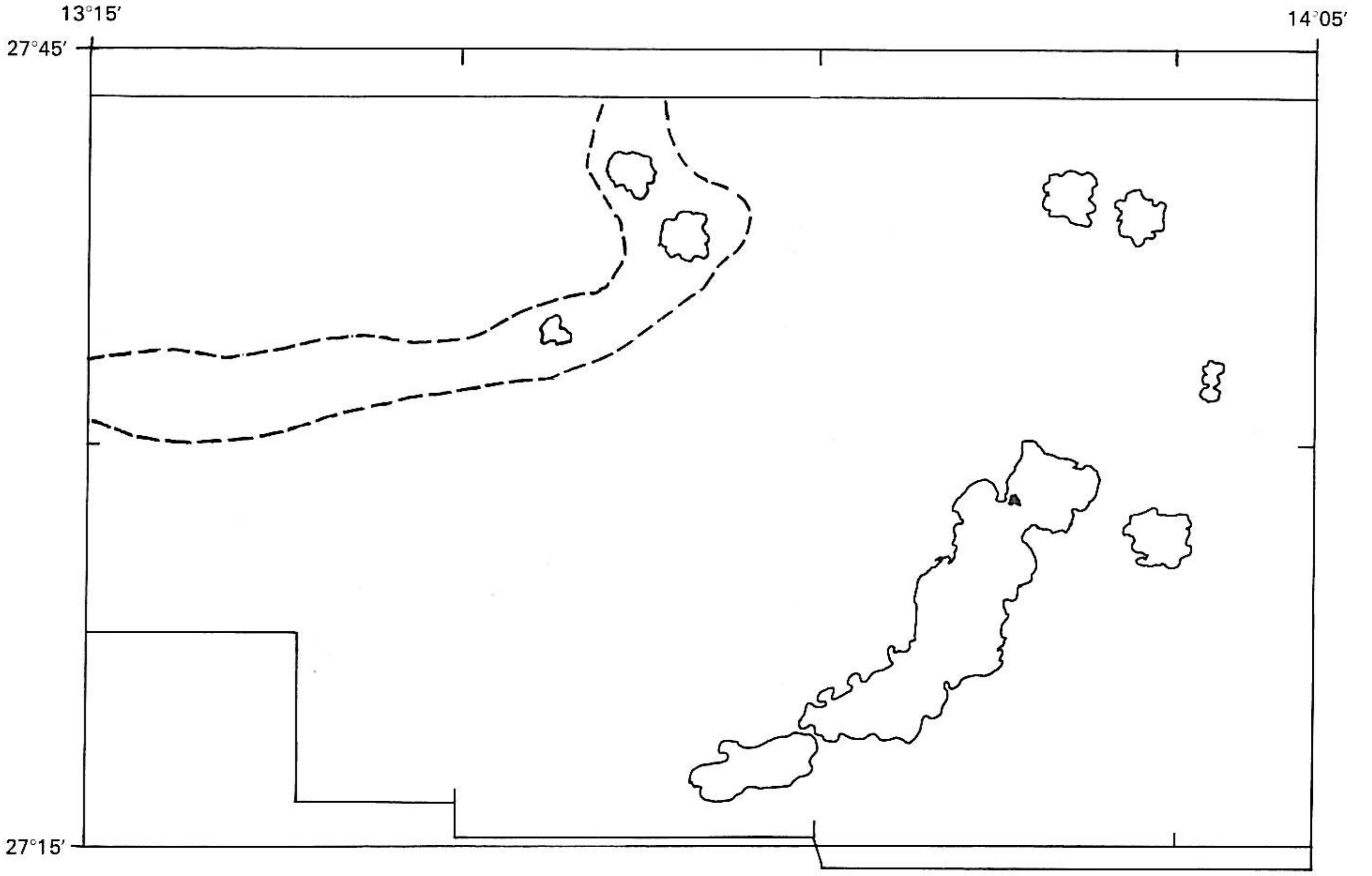

FIG. 4. Schematic outline of the anomalous zone (continuous line) and newly delineated zone (dashed line).

The enhanced signals $(M \times A)$ and $(M \times H)$ of two profiles (L-258) and (L-236) which pass over the drill holes are shown in Figure 2.

Figure 2 indicates that the signal resolution is quite predominant along the profiles over the drill holes.

The enhanced magnetic signals along profiles (L-111–L-195) delineate an anomalous zone extending north–northwest, where the pre-enhanced signals are very weak. The outline of the boundary of this zone is schematically shown in Figure 4.

As an example of the enhanced signatures outlining this zone, two profiles are shown in Figure 3. It is, also, important to note that the enhanced magnetic signatures over these profiles, although predominant, they are relatively low in magnitude compared to the profiles on the south eastern side, where the known anomalous zone is almost exposed near the surface (see Figure 2 and 3). This suggests that the new anomalous zone is deep seated and covered by the sand in the region.

It is pertinent to mention, also, that the analysis of the aeromagnetic data indicates a few patterns of enhanced signals at various locations. These signals appear to follow the structural features like faults and contacts, in the region.

The analysis of the filtered residual total aeromagnetic data using the Hilbert transform, not only enhanced the signals, verifying the known anomalous zone in the area, but also enabled the delineation of new zone trending north–northwest.

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REFERENCES

Cerney, V. and Zahradrik, K.J., 1975, Hilbert transform and its geophysical application, Acts. Uni. Math. Phys. 16, 67–81.

Erdelyi, A., and Magnus, W., *et al.*, 1954, Table of Integral transforms, Vol. II, McGraw-Hill Book Co., Inc.

Goudarzi, G.H., and Tschoepke, Dr. Ing. R., 1968, A geologic report on the iron deposit of the Shatti Valley area of Fezzan, Libya; Geological Section Bulletin No. 7, p. 9–10, Ministry of Industry, Libya.

Mohan, N.L., Rama Dass, G., and Varaprasada Rao, S.M., 1985, Resolution of gravimagnetic and radiometric regional anomalies of Cuddapah basin, A.P., India, using the Hilbért transform; accepted for European Association of Exploration Geophysicists Conference, Budapest, Hungary. Printed in the Abstracts booklet.

Mohan, N.L., Rama Dass, G., Varaprasada Rao, S.M., and Sunderarajan, N., 1988, Hilbert transform analysis for gravimagnetic and radiometric regional anomalies of Cuddapah basin, A. P., India – A qualitative study: Accepted for publication in Pure and Applied Geophysics

Mohan, N.L., Sunderarajan, N., and Seshagiri Rao, S.V., 1982, Interpretation of some two dimensional magnetic bodies using the Hilbert transform, Geophysics, 47, 376–387.

Nabighian, M.N., 1972, The analytic signal of two dimensional magnetic bodies with polygonal cross section, its properties and use for automated anomaly interpretation, Geophysics, 37, 507–512.

Rama Dass, G., Varaprasada Rao, S.M., and Mohan, N.L., 1986, Application of the Hilbert Transform in resolution of magnetic signals – A new technique: Australian Bulletin of Exploration Geophysicists, Vol. 17.

Shimshoui, M. and Smith, S. W., 1964, Seismic signal enhancement with three component detectors, Geophysics 29, 664–671.

Shuey, R.T., 1972, Application of Hilbert transforms to magnetic profiles: Geophysics, Vol. 37, No. 6, 1043–1045.

Thomas, J.B., 1969, An introduction to statistical communication theory, 639–657, John Wiley, New York.