PRJ, Vol. 6 (January 1994); P. 44–51, 9 FIGS.

HILBERT TRANSFORM AND IMAGE SHARPENING TECHNIQUES AS A TOOL OF INTERPRETING 3-D POTENTIAL FIELD (GRAVITY) DATA

A.M. Ushah*

استخدام محول هلبرت وتقنيات وضوح الصورة في تفسير معلومات الجاذبية

عبد الرزاق محمد أوشاح

تعرض هذه الورقة دراسة تحليلية وعددية بطريقة محول هلبرت ذي البعدين وتطبيقه على معلومات الجاذبية من مصادر (أجسام) ذات ثلاثة أبعاد.

هذا وقد تم التطرق إلى العلاقات الرياضية التي تمثل محول هلبرت كدالة في الزمن وكدالة في التردد، وكذلك العلاقات الرياضية التي تمثل شذوذ الجاذبية للمصادر تحت السطح، بالإضافة إلى بعض العلاقات الأخرى مثل مرشحات المرور العالية المثالية، المشتقة الأولى والثانية سواء كانت الرأسية أو الأفقيتين لشذوذ الجاذبية.

إن استخدم تقنيات وضوح الصورة (Image sharpening) للمشتقة الرأسية الثانية، لشذوذ الجاذبية، الناتجة عن مجموعة مصادر ذات أسطح مستطيلة متوازية، يعتبر مفيد جداً في تتبع ومعرفة حواف وحدود هذه المصادر.

ABSTRACT

The paper presents an analytical and numerical study of two-dimensional (2-D) Hilbert transform method as applied to the three-dimensional (3-D) potential field (gravity) data for interpretation and image sharpening. The relations of two-dimensional (2-D) Hilbert transform in time and frequency domains are presented and a numerical algorithm package has been developed.

Application of 2-D Hilbert transform in quantitative interpretation of gravity anomalies due to multiprismatic bodies as well as their derivatives are investigated and used to test the 2-D Hilbert transform technique. The results achieved by using this relatively new technique are excellent.

Image sharpening techniques of the second vertical derivative of gravity effect due to multiprismatic bodies are useful primarily as an enhancement tools for highlighting edges in an image. These techniques can be achieved either by differentiation or by highpass filtering.

* Petroleum Research Centre, P.O. Box 6431, Tripoli, G.S.P.L.A.J.

INTRODUCTION

This paper deals with the application of 2-D Hilbert transform, to problems associated with the interpretation of three dimensional potential field (gravity) data, and image sharpening (as enhancement tools).

The 2-D Hilbert transform expressions were derived by Bose and Prabhu (1974) in several forms such as cotangent, sine and matrix forms whereas Nabighian (1984) used 2-D signum function.

A closed expression developed for the gravitational attraction of a prism was derived by Nagy (1966) which is valid for any point outside of or on the boundary of the prism, and this was later modified by Goodacre (1973). This expression is extended to multiprismatic bodies and simplified by Ushah (1988).

The discussion is extended to image sharpening of the second vertical derivative in the spatial and frequency domains by differentiation and by highpass filtering respectively. In the spatial domain, the theory of the gradient approach is derived, which is most commonly used in image processing applications as an enhancement tool for edge detection. In frequency domain, the ideal highpass filter, which attenuates the low-frequency components without disturbing high-frequency components in the Fourier, transform is discussed.

THEORY OF HORIZONTAL AND VERTICAL DERIVATIVES OF A 3-D POTENTIAL FUNCTION AND 2-D HILBERT TRANSFORM

The 2-D Hilbert transform can be expressed in several forms, such as cotangent, sine and matrix forms (Bose and Prabhu, 1979). The 2-D Hilbert transform operator in cotangent form (Fig. 1) is given below as:

$$ht(i,j)=\frac{2}{N_1N_2}\cot\frac{\pi}{N_1}i+\frac{2}{N_1N_2}\cot\frac{\pi}{N_2}j \quad (1)$$
$$=ht_1+ht_2$$

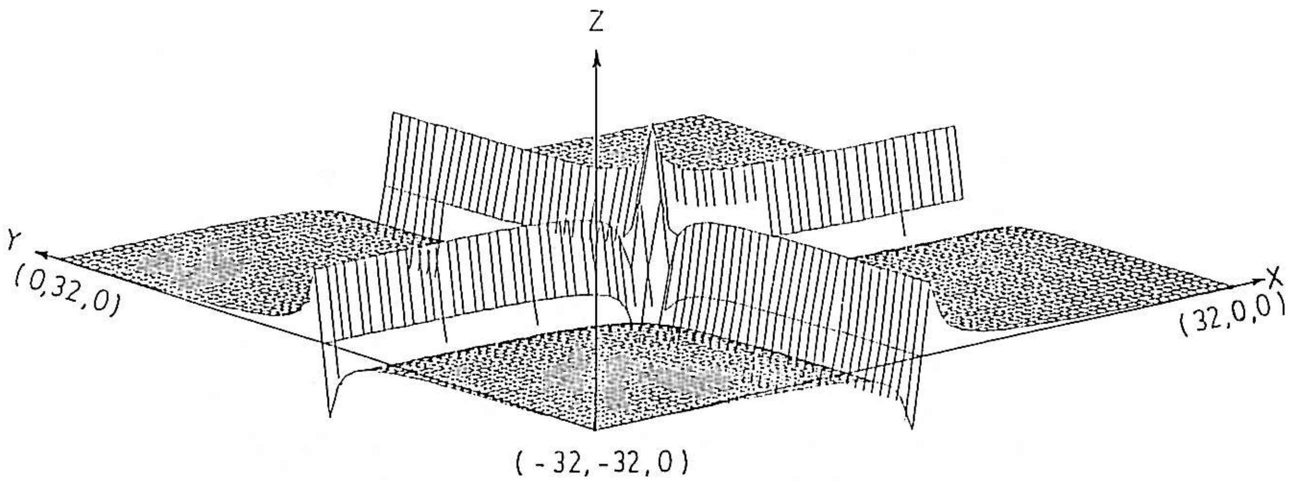

FIG. 1. Two dimensional Hilbert transform as obtained by using the cotangent form (equation 1).

where

$$ht_1=\frac{2}{N_1N_2}\cot\frac{\pi}{N_1}i,\quad ht_2=\frac{2}{N_1N_2}\cot\frac{\pi}{N_2}j$$
$$(i=0,1,2,\ldots,N_1-1)$$
$$(j=0,1,2,\ldots,N_2-1)$$

In frequency domain, the 2-D Hilbert transform can be expressed as (Nabighian, 1984):

$$H(p,q)=-j\,\mathrm{sgn}(p,q)=H_1\widehat{ex}+H_2\widehat{ey} \quad (2)$$

where

$$\mathrm{sgn}(p,q)=[p/(p^2+q^2)^{1/2}]\widehat{ex}+[q/(p^2+q^2)^{1/2}]\widehat{ey}$$
$$(j=(-1)^{1/2})$$

$\widehat{ex}$ and $\widehat{ey}$ are the unit vectors in the x and y directions, and

$$H_1=-jp/(p^2+q^2)^{1/2} \quad \text{and} \quad H_2=-jq/(p^2+q^2)^{1/2}.$$

If M is a 3-D potential function, then, the relationship between the horizontal and vertical derivatives of M and the 2-D Hilbert transform may be written as:

$$F[\mathrm{d}M/\mathrm{d}z]=H_1F[\mathrm{d}M/\mathrm{d}x]+H_2F[\mathrm{d}M/\mathrm{d}y] \quad (3)$$

$$F[\mathrm{d}^2M/\mathrm{d}z^2]=H_1F[\mathrm{d}^2M/\mathrm{d}z\mathrm{d}x]+H_2F[\mathrm{d}^2M/\mathrm{d}z\,\mathrm{d}y] \quad (4)$$

where F is the Fourier transform, $\mathrm{d}M/\mathrm{d}z$ is the vertical derivative of M, $\mathrm{d}M/\mathrm{d}x$ and $\mathrm{d}M/\mathrm{d}y$ are the horizontal derivatives of M, in x and y directions, respectively, and $\mathrm{d}^2M/\mathrm{d}z^2$ is the second vertical derivative of M.

It is worth mentioning at this point that the numerical computation of equations (3) and (4) are stable because both H_1 and H_2 are well behaving operators with values not exceeding ± 1.

GRAVITATIONAL ATTRACTION OF MULTIPRISMATIC BODIES

The expression for the gravitational attraction of multiprismatic bodies at a point of observation x, y and z can be written as:

$$Gt(x,y,z)=\gamma\sum_{t=1}^{N}\rho_t(br_{t_1}+br_{t_2}+br_{t_3}) \quad (5)$$

where $Gt(x,y,z)$ is the vertical attraction of t prisms at a point (x,y,z), ρ is the density of the prism, N is the number of the respective prism, and γ is the gravitational constant

$$br_{11} = \left|\left|\left| u \operatorname{arcsinh}[v/u^2 + w^2)^{1/2}] \right|_{x-u_1}^{x-u_2}\right|_{y-v_1}^{y-v_2}\right|_{z-w_1}^{z-w_2}$$

$$br_{12} = \left|\left|\left| v \operatorname{arcsinh}[u/v^2 + w^2)^{1/2}] \right|_{x-u_1}^{x-u_2}\right|_{y-v_1}^{y-v_2}\right|_{z-w_1}^{z-w_2}$$

$$br_{13} = \left|\left|\left| -w \arctan[v/w(u^2 + v^2 + w^2)^{1/2}] \right|_{x-u_1}^{x-u_2}\right|_{y-v_1}^{y-v_2}\right|_{z-w_1}^{z-w_2}$$

In equation (5), the terms in each bracket correspond to a particular prism (Ushah, 1988).

SOME EXAMPLES OF GRAVITATIONAL ATTRACTION OF MULTIPRISMATIC BODIES

The vertical component of the gravity field due to five prismatic bodies Fig. 2 is obtained by using equation 5. The horizontal sides of the Prism 1 extend 14 by 14 pixels and its center at (32, 32) pixel. Prism 2 has 12 pixels length in x and y directions and its center at (16, 48) pixels. Prism 3 has 2 units less than Prism 2 in each horizontal dimension and its center at (48, 16) pixel. Prism 4 and Prism 5 have horizontal dimensions 8 by 8 and 6 by 6 units and their centers at (16, 16) and (48, 48) pixels, respectively. The depths to the top of all the prisms in this model is equal 2 units below the surface of observation. Figures 3 and 4 represent the first and second vertical derivatives obtained by using equations 3 and 4 respectively. From these figures the boundaries of the prisms in the model are very clear and can be determined easily especially from the second vertical derivative (Fig. 4).

Figures 5 and 6 are other examples which are tested by increasing the depths of all the prisms to 5 units below the surface of observation.

IMAGE SHARPENING

In this section two methods of sharpening techniques (Gonzales and Wintz, 1977) are discussed, which can be considered as enhancement tools to outline the boundary for the subsurface bodies. They

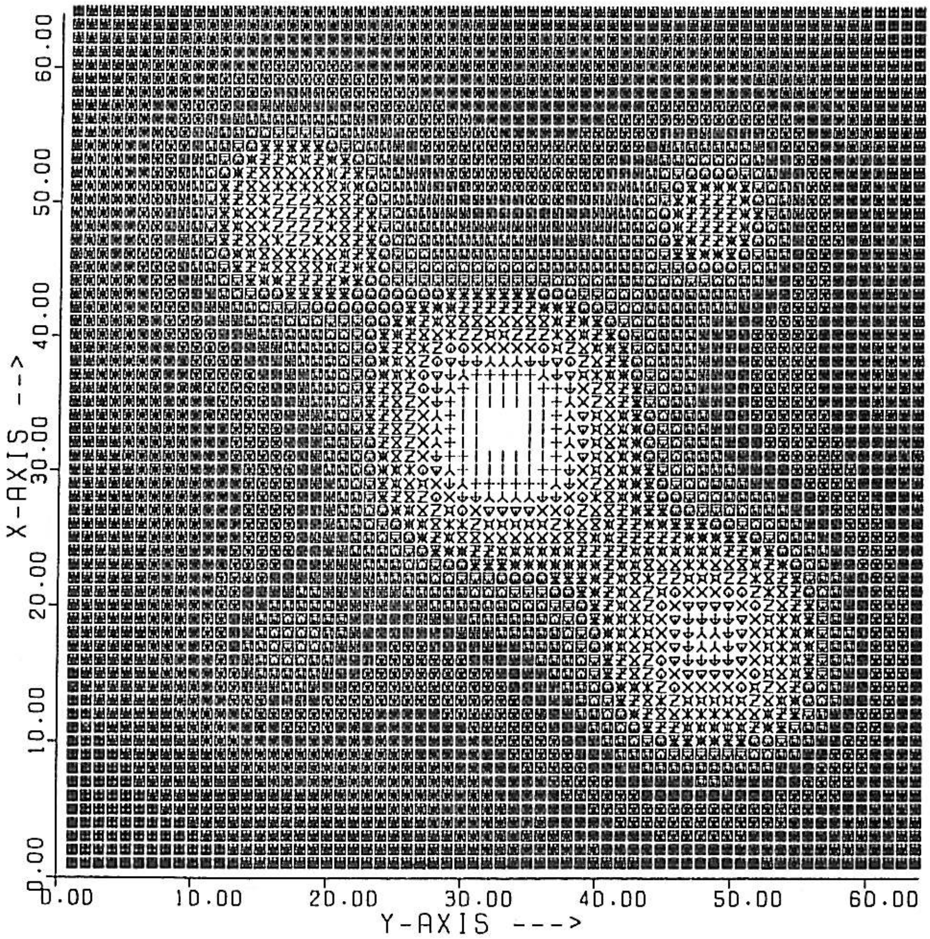

FIG. 2. Example of theoretical gravity effect due to five prismatic bodies; the image displayed with 64 by 64 pixels. $h_1 = 2.0$ for all prisms.

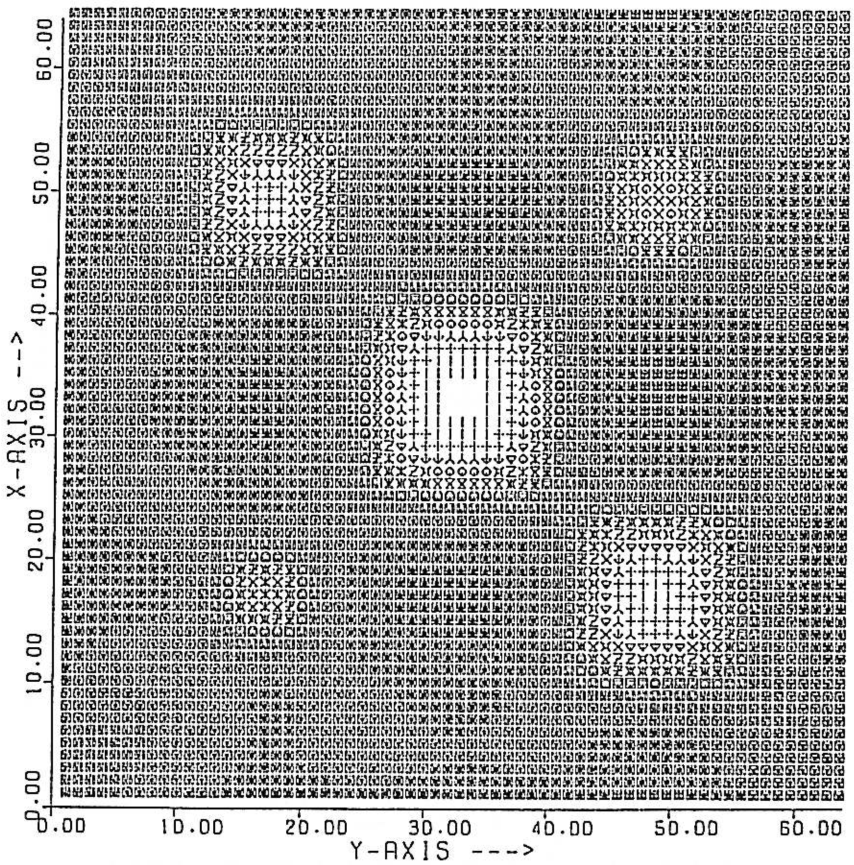

FIG. 3. First vertical derivative of the gravity effect obtained by using 2-D Hilbert transform.

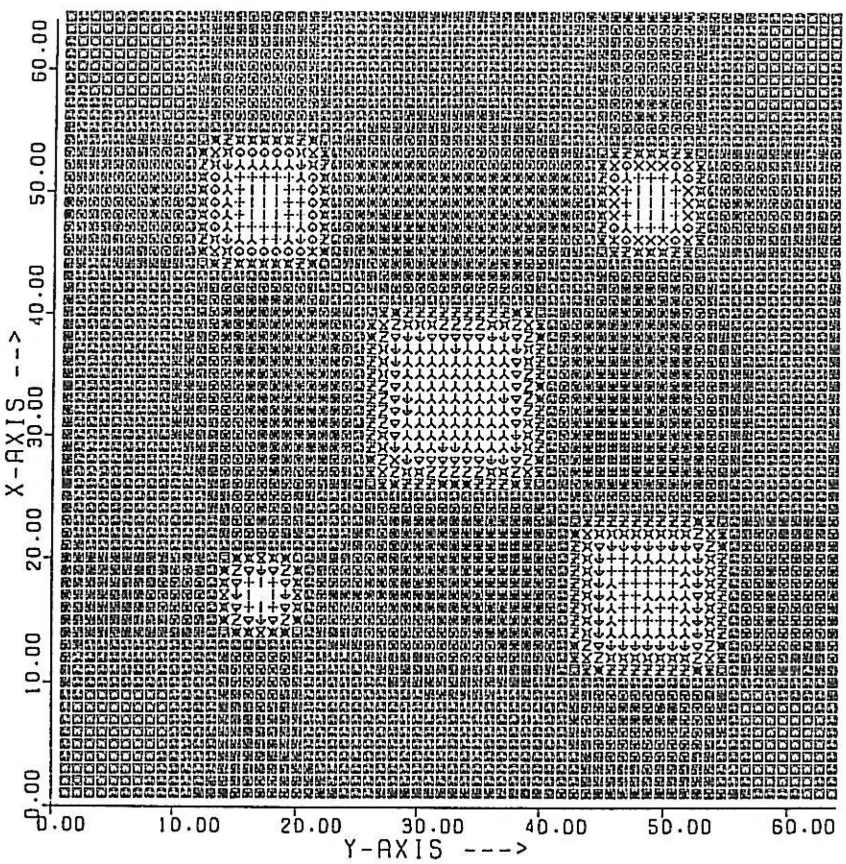

FIG. 4. Second vertical derivative of the gravity effect obtained by using 2-D Hilbert transform (equation 4).

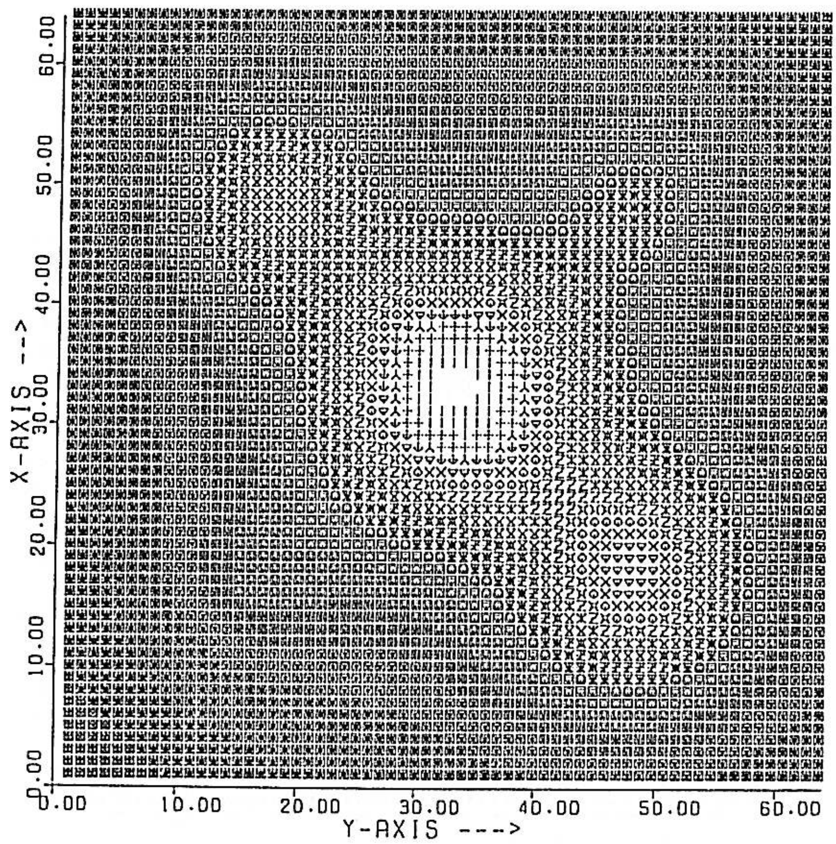

FIG. 5. Example of theoretical gravity effect due to five prismatic bodies; the image displayed with 64 by 64 pixels. h_1(depth) = 5.0 for all prisms.

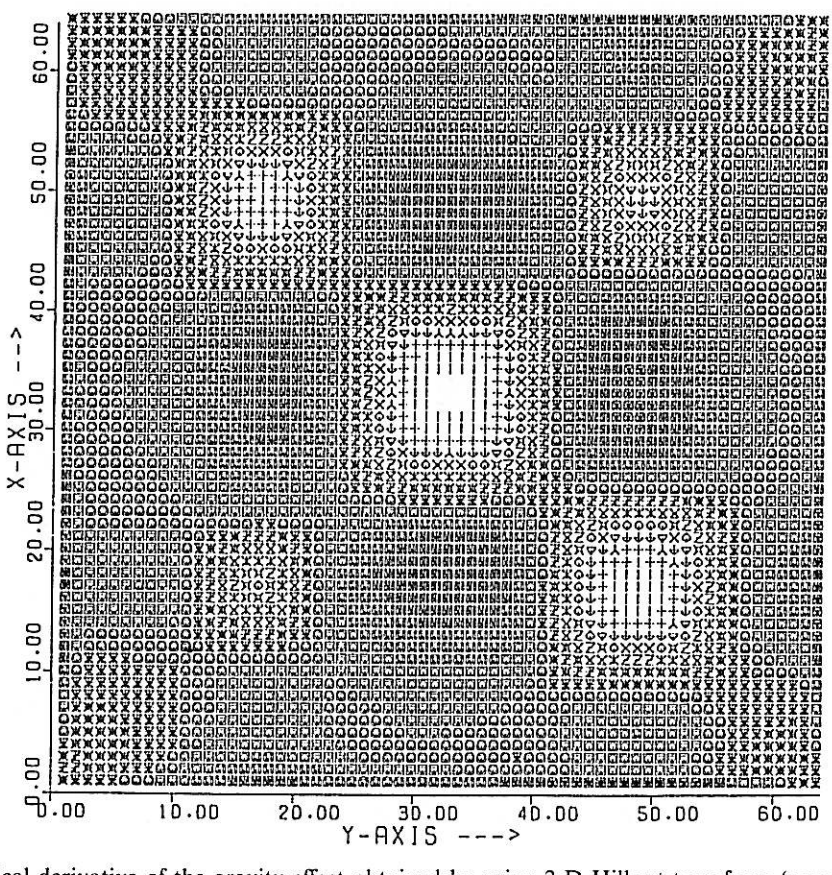

FIG. 6. Second vertical derivative of the gravity effect obtained by using 2-D Hilbert transform (equation 4).

are sharpening by differentiation and highpass filtering.

Sharpening by Differentiation

The most commonly used method of differentiation in image processing application is the gradient ($G[f(x, y)]$), which can be expressed as:

$$G[f(x, y)] = \text{mag}[G] = \left[\left(\frac{df^2}{dx} + \frac{df}{dy}\right)^2\right]^{1/2} \tag{6}$$

For a digital image, the derivatives in equation 6 are approximated by differences. One typical approximation is given by the relation

$$G[f(x, y)] = [[f(x, y) - f(x+1, y)]^2 + [f(x, y) - f(x, y+1)]^2]^{1/2} \tag{7a}$$

Similar results are obtained by using absolute values, as follows:

$$G[f(x, y)] = |f(x, y) - f(x+1, y)| + |f(x, y) - f(x, y+1)| \tag{7b}$$

This formulation is more desirable for a computer implementation of the gradient.

The disadvantage of equations 6 and 7 is that all smooth regions in $f(x, y)$ appear dark in $G[f(x, y)]$ because of the relatively small values of the gradient in these regions. One solution to this problem is to form a new function $g(x, y)$ as follows:

$$g(x, y) = \begin{cases} G[f(x, y)] & \text{if } G[f(x, y)] \geqslant T \\ f(x, y) & \text{otherwise} \end{cases} \tag{8}$$

where T is a nonnegative threshold. By selecting suitable value for T, it is possible to emphasize significant edges without obsecuring the characteristics of the smooth backgrounds.

Finally, if only the location of edges is of interest, the relation becomes

$$g(x, y) = \begin{cases} LG & \text{if } G[f(x, y)] \geqslant T \\ LB & \text{otherwise} \end{cases} \tag{9}$$

where LG and LB are specified gray levels for edges and the background.

The properties of the gradient are relatively large for the prominent edges in an image, and small values in regions that are fairly smooth, being zero only in

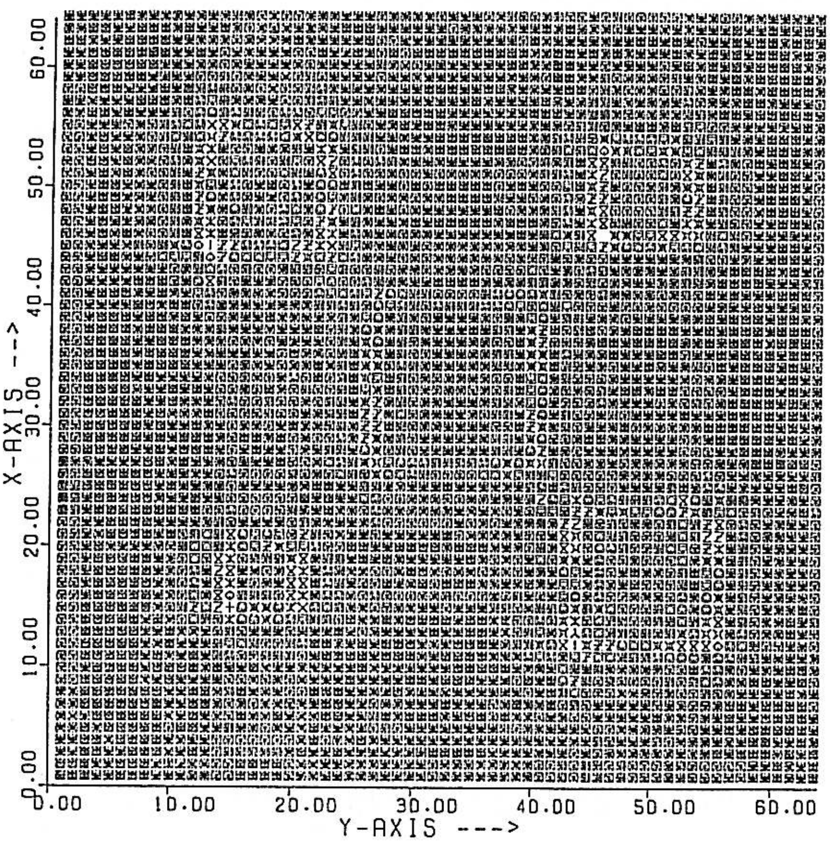

FIG. 7. Example of image sharpening by differentiation of Fig. 4. Illustration of edge enhancement by gradient techniques (equation 7b).

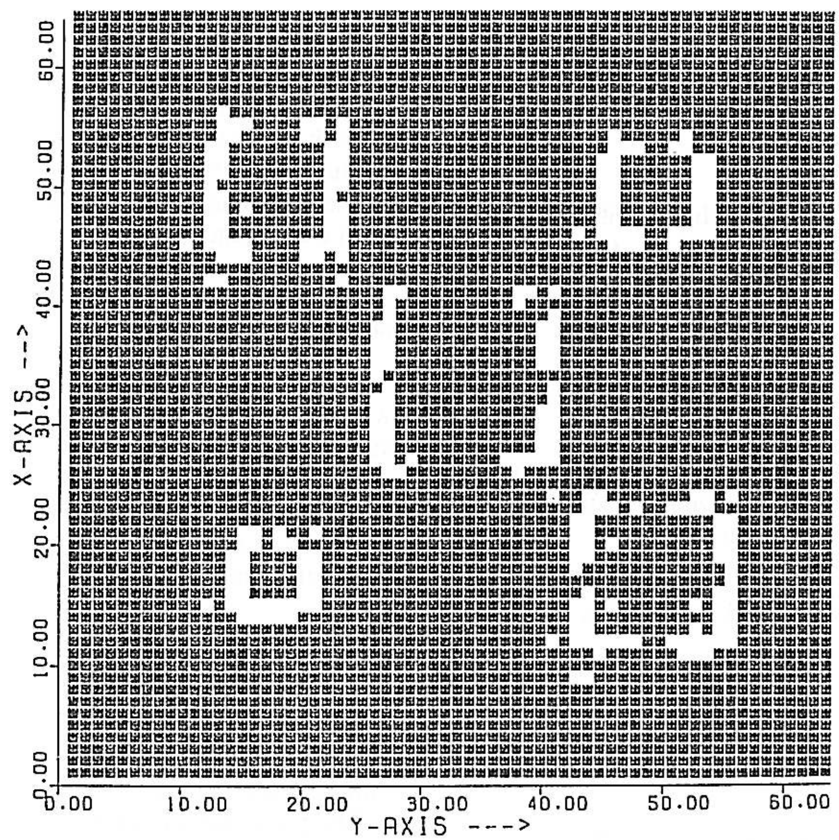

FIG. 8. Represents the edge effect detection (equation 9) with a nonnegative threshold $(T)=10$.

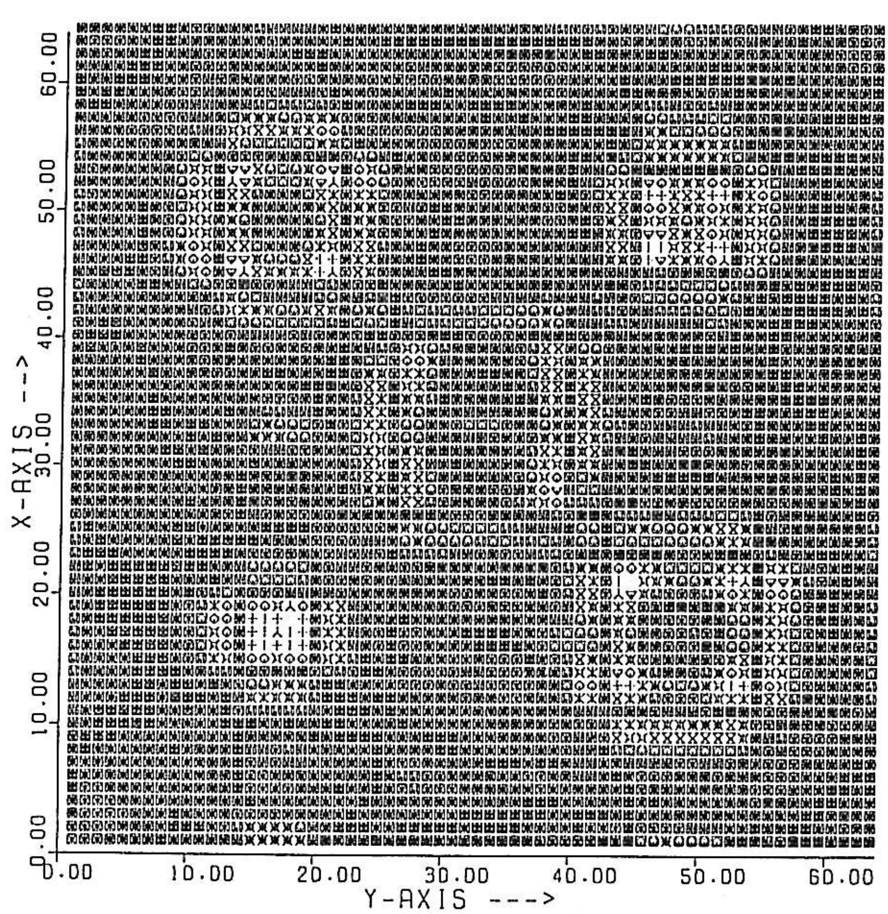

FIG. 9. Image sharpening achieved in the frequency domain by highpass filtering.

regions with constant gray level. These properties are illustrated in Figs. 7, 8 and 9. As shown in Fig. 7, the gradient operation (equation 7b was used) reduces all the constant white regions to zero (black), leaving only the points associated with abrupt changes in gray level (in this case the edge boundaries of the prisms).

Figure 8 is the result of using the gradient scheme given by equation 9 with $T=10$, $LG=32$ and $LB=1$. We notice that considerable amount of small segments appear in the resulting image, with the strongest intensities taking place around the border of the prisms. This is an expected result since the magnitude of the gradient is proportional to changes in gray levels and should be more prominent in regions of an image containing distinct edges.

Highpass Filtering

It is well known that an image can be blurred by attenuating the high-frequency components. Since the high-frequency components can be detected on the edges and other abrupt changes in gray levels, therefore, image sharpening can be achieved in frequency domain by a highpass filtering process which attenuates the low frequency components without disturbing high-frequency components.

Ideal Filter

The transfer function which satisfies the following relation is called a two-dimensional ideal highpass filter (IHPF);

$$H(u, v)=\begin{cases}0 & \text{if } D(u, v)\leqslant Do\\ 1 & \text{if } D(u, v)>Do\end{cases} \qquad (10)$$

where Do is the cut-off distance measured from the origin of the frequency plane, and $D(u, v)=(u^2+v^2)^{1/2}$.

Figure 9 shows the sharpening in the image achieved by applying IHPF with $Do=\sqrt{10}$. The edges of the prisms are predominant only in this image because the low frequency components were severely attenuated.

CONCLUSION

The 2-D Hilbert transform can be considered as one of the most important trasforms in image processing for image enhancement.

The first and second vertical derivatives of the gravity field caused by multiprismatic bodies can be obtained by 2-D Hilbert transform technique. Such techniques are used to locate the vertical projection of the subsurface bodies.

The image sharpening of the second vertical derivative obtained either by differentiation or highpass filtering, can be used as an additional tool to locate the boundary of the subsurface bodies.

The results indicate that the discussed techniques are quite useful in isolating the anomalies in 3-D potential field data.

ACKNOWLEDGEMENT

The author would like to acknowledge the support and encouragement of the Petroleum Research Centre received during the preparation of this work.

REFERENCES

Bose, N.K. and Prabhu, K.A., 1979, Two-Dimensional Discrete Hilbert Transform and Computational Complexity Aspects in its Implementation, IEEE Transactions on Acoustics, Speech and Signal processing, Vol. Assp-27, No. 4.

Curts, F.G., 1980, Applied Numerical Analysis, Addison-Wesley Publishing Co.

Gonzales, R.C. and Wintz, P., 1977, Digital Image Processing, p. 154–162, 183–211.

Goodacre, A.K., 1973, Some Comments on the Calculation of the Gravitational and Magnetic Attraction of a Homogenous Rectangular Prism, Geophysical Prospecting, Vol. 21, p. 66–68.

Hall, E.L., 1979, Computer Image Processing and Recognition, p. 239–242.

Nabighian, M.N., 1984, Toward a Three-Dimensional Automatic Interpretation of Potential Field Data Via Generalized Hilbert Transforms: Fundamental Relations, Geophysics, Vol. 49, No. 6, p. 780–786.

Nagy, D., 1966, The Gravitational Attraction of a Right Rectangular Prism, Geophysics, Vol. 21, p. 362–371.

Read, R.R. and Trietel, S., 1973, The Stabilization of Two-Dimensional Recursive Filters Via the Discrete Hilbert Transform, IEEE Transactions on Geoscience Electronics, GE-11, p. 153–160.

Wooil, M.M., Ushah, A., Singh, V., and Bruce, B., 1988, Application of 2-D Hilbert Transform in Geophysical Imaging with Potential Field Data. IEEE Transaction on Geoscience and Remote Sensing, Vol. 26, No. 5, p. 502–510.