PRJ, Vol. 6 (January 1994); P. 1–8, 5 FIGS, 3 TABLES.

RESTRICTION CONDITIONS IN WELL LOG INTERPRETATION

G. Szigeti*

الظروف المقيدة لاستقراء سرود الآبار

ج. زيقيتي

عند مقارنة السرود المقاسة والمستنبطة في عملية الاستقراء لا بد وأن تستوفى معايير الصخور بعض الظروف المحددة.

تبين هذ الورقة كيفية تناول الظروف المقيدة بحصر التعريف في نطاق صلب محذب (متعدد السطوح) على رقعة مساحية كبيرة. وتقدم هذه الورقة حلا رياضيا ولوغارتميا للتقليل من عدم الترابط بين مجموعتين من السرود على هذا النطاق الصلب المحدب.

وتثبت الدراسة صحة عملية التقليل هذه وتوضحها على مثال هندسي حي كما توصف أسس التجسيم الحاسوبي للوغارتم. كذلك تم تبيان الظروف المقيدة المختلفة للإستقراد بالصخور الجرانيتية.

ABSTRACT

In comparing the measured and response logs during the process of complex well log interpretation, the parameters of the rock model must fulfill certain restriction conditions.

The present paper shows how the different types of restriction conditions can be handled by constraining the domain of definition on a convex solid (Polyhedron) of higher dimensional space. The paper offers and realizes complete mathematical and algorithmic solution for the minimization of incoherency between the two set of logs on this convex solid. The paper proves the correctness of the process of minimization, demonstrates it on a vivid, geometrical example and describes the basic concepts of computer realization of the algorithm.

Application of different restriction conditions for an interpretation in granite rock is also given.

INTRODUCTION

One of the main aspects of the complex well log interpretation is the comparison of the measured logs and the response logs. The latter one is created by the log analyst on the basis of geological, petrophysical knowledge using the rock model of the area and the response functions of the measurements. The aim of the comparison is the determination of the parameters of the rock model.

The parameters undergo certain restrictions such as any fraction of unit volume is between zero and one. For most wells and for most log intervals when we have a successful trial of methodology, instrument and interpretation, these restrictions are fulfilled automatically (i.e. without any special mathematical arrangement during the process of minimization of the difference between measured and theoretical logs). After all, if because of the erroneous character of the measurements or because of approximate character of rock model the result of minimization breaks the restrictions it does not necessarily mean that the given field material could not be explained on the basis of the used rock model. This question finally must be judged on the basis of the measure of incoherency between the two quantities. The aim of the computerized interpretation should be to gain all the useful information from the field material. That is why in the case of the above mentioned difference we

*Petroleum Research Centre, P.O. Box 6431, Tripoli, G.S.P.L.A.J.

have to form and solve a more special mathematical minimization task.

From purely mathematical point of view the task elaborated in the present paper is the minimization of a quadratic function on a convex body of k-dimensional Euclidian space, where the body is a polihedron, convex hull of a finite point set. The offset of the existing unique minimum is to be found by the aid of a series of projections of different dimensions. With growing k, the number of projections that might be necessary grows very fast in a non polynomial order.

In bore hole geophysics we have only a limited number of adequate logs, thus the number of parameters to be determined could not be too high. Consequently there is a strict connection between the applied mathematical solution and the geophysical task: our algorithm is fully applicable, even in a relatively modest computer, until four, five, six unknowns, which cover the needs of loganalysts. This is, definitely, inapplicable if the number of unknowns is more than ten. On this price we get a rather exact solution of the problem of restrictions without the necessity of introducing penalty term containing subjective construction.

TYPE OF RESTRICTIONS

Suppose we have m different measured log values at a given fixed depth point, each one is normalized by its own standard deviation, $\boldsymbol{b}^{\mathrm{T}}=(b_1, b_2, \ldots, b_m)$, and suppose that we have already accepted that our rock model consists of one fluid and k solid unknown components $\boldsymbol{x}^{\mathrm{T}}=(x_0, x_1, \ldots, x_k)$, where $k \leqslant m$ and $\boldsymbol{x}^{\mathrm{T}}$ is the transposed row vector of the column vector $\boldsymbol{x}$. We approach each tool response function in a linear form using the normalized, zone parameter matrix A having m rows and $k+1$ columns.

Our aim is to find the parameter vector $\boldsymbol{x}$ for which $A \cdot \boldsymbol{x}$ is the possible closest to the measured vector $\boldsymbol{b}$ on the set of Euclidian space $Q \subset \mathbb{R}^{k-1}$:

$$\hat{x}_Q=\{\boldsymbol{x} \in Q \mid \| \boldsymbol{b}-A\boldsymbol{x} \|^2 \rightarrow \min\}, \tag{1}$$

where $\| \ \|^2$ is the square of the distance in the $\mathbb{R}^m$ vector space.

The simplest condition for the set Q is given by the material balance equation according to which the sum of all the components is unity (we involve this equation into the definition of set Q instead of adding one more term to the function to be minimized, because by this way we fulfill the material balance equation exactly and not only approximately).

We speak about interpretation *without restrictions* if we define set Q as

$$Q=Q_{nr}=\{\boldsymbol{x} \in \mathbb{R}^{k+1} \mid x_0+x_1+\cdots+x_k=1\} \tag{2}$$

Definition (2) does not guarantee that all x_i are positive, or x_i are less than one, i.e. $0 \leqslant x_i \leqslant 1$ is not necessarily valid for all x_i. If we prescribe together with the material balance equation that $0 \leqslant x_i$ for $i=0, 1, \ldots, k$ then it trivially follows that $x_i \leqslant 1$ $(i=0, 1, \ldots, k)$, thus we speak about *logical restrictions* when we define Q as

$$Q=Q_{lr}=\left\{\boldsymbol{x} \in \mathbb{R}^{k+1} \,\middle|\, \begin{array}{l} 0 \leqslant x_i, (i=0, 1, \ldots, k) \\ x_0+x_1+\cdots+x_k=1 \end{array}\right\} \tag{3}$$

It should be noted here, that if we project points of Q_{lr} to the $(0, x_1, x_2, \ldots, x_k)$ hyperplane of $\mathbb{R}^{k+1}$, then the projection is the k-dimensional simplex of the $(x_1', x_2', \ldots, x_k')$ $\mathbb{R}^k$ space, given by the points $P_0=(0, 0, \ldots, 0)$; $P_1=(1, 0, 0, \ldots, 0)$; ...; $P_k=(0, 0, \ldots, 1)$. This projection allows inversion in the form $x_0=1-x_1'-x_2', \ldots, -x_k'$; $x_1=x_1'$; $x_2=x_2'$; ...; $x_k=x_k'$, and in this sense we say that Q_{lr} is a k-dimensional simplex (embedded into the $k+1$-dimensional space).

In definition (3) of Q_{lr} we speak about logical restrictions, because this allows to change x_i in the wider logically possible interval. Another situation is when e.g. from core lab data we can prescribe more strict limits for certain components x_i. In connection with this, we speak about *geological restrictions* if we define Q as

$$Q=Q_{gr}=\left\{\boldsymbol{x} \in \mathbb{R}^{k+1} \,\middle|\, \begin{array}{l} a_i \leqslant x_i \leqslant b_i, (0 \leqslant a_i < b_i \leqslant 1), \\ (i=0, 1, \ldots, k) \\ x_0+x_1+\cdots+x_k=1 \end{array}\right\} \tag{4}$$

First we notice that definition (4) contains no contradiction, i.e. set Q_{gr} is nonempty, if and only if

$$\sum_{i=0}^{k} a_i \leqslant 1 \quad \text{and} \quad 1 \leqslant \sum_{i=0}^{k} b_i \tag{5}$$

Second let's examine again the $\boldsymbol{x}=(x_0\, x_1, \ldots, x_k) \rightarrow (0, x_1, x_2, \ldots, x_k)=(x_1', x_2', \ldots, x_k')=\boldsymbol{x}'$ projection of the set Q_{gr}. Denoting the projection of Q_{gr} by Q_{gr}', it is not difficult to realize that Q_{gr}' can be represented in the following form:

$$Q_{gr}'=T_k \cap H_{1-b_0, 1-a_0}, \tag{6}$$

where

$$T_k=\{\boldsymbol{x}' \in \mathbb{R}^k \mid a_i \leqslant x_i' \leqslant b_i, (i=1, 2, \ldots, k)\}, \tag{7}$$

$$H_{c,d}=\left\{\boldsymbol{x}' \in \mathbb{R}^k \mid c \leqslant \sum_{i=1}^{k} x_i' \leqslant d\right\} \tag{8}$$

which means that Q_{gr}' is the intersection of the k-dimensional rectangular parallelepiped T_k and the

$H_{c,d}$ strip like domain $\mathbb{R}^k$ (domain bounded by two parallel hyperplanes) with parameters $c=1-b_0$, $d=1-a_0$. Figure 1 demonstrates different possible Q'_{gr} restriction sets for the $k=2$ dimensional case, when we have two solid components in our rock model.

Q_{lr} evidently $n_{max}=k+1$. The comparison of these two expressions of n_{max} explains how more complicated the task becomes for the case of geological restrictions, and the reason for restricting our solution to lower dimensions.

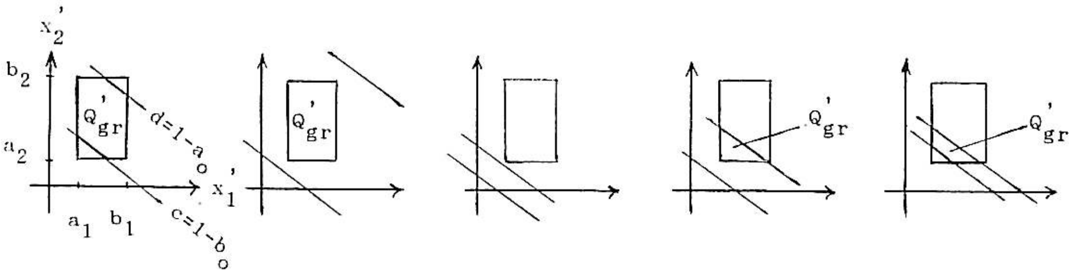

FIG. 1. Different restriction sets for two solid components model.

The projection above allows inversion in the form $x'=(x'_1, x'_2, \ldots, x'_k) \to (1-\Sigma_{i=1}^{k} x'_i, x'_1, x'_2, \ldots, x'_k) = (x_0, x_1, \ldots, x_k) = x$ and in this sense we say that Q_{gr} is a k-dimensional polyhedron (embedded into the $k+1$ dimensional space).

It is not difficult to realize, that Q_{nr}, Q_{lr} and Q_{gr} are convex sets, i.e. from $x_0, x_1 \in Q$ it follows that $\alpha x_0 + (1-\alpha)x_1 \in Q$ for any $0 \leqslant \alpha \leqslant 1$.

During the process of minimization a pivotal point is the characterization of the Q sets by his vertexes, i.e. representation of Q as the convex hull $\kappa(P_1, P_2, \ldots, P_n)$, which is the narrowest convex set containing all the P_i points. For Q_{nr} this representation is not possible and not necessary, for Q_{lr} we have already described the vertexes of the simplex. Let's find the vertexes of the Q_{gr} set. As the representation (6) of Q_{gr} shows in order to reach this aim we have to examine whether the jth edge of the rectangular parallelepiped T_k is completely inside, completely outside or intersects once or twice the strip $H_{1-b_0,1-a_0}$. The only difficulty at this stage is to create an algorithm, which runs over all the edges of the rectangular parallelepiped.

For the sake of simplicity let's examine instead of block T_k the equivalent in this respect to the unit cube. The k-dimensional unit cube has $k \cdot 2^{k-1}$ edges, and two peak points of the cube are connected by edge if and only if there is exactly one coordinate where the two peak points differ. Figure 2 shows the edge system of the unit cube of $\mathbb{R}^4$. Ordering the peak points of Fig. 2 from up to down and if the two points are on the same level then from left to right and directing edges from the smaller to the higher points, i.e. $\overrightarrow{P_iP_j}$ is directed edge if $\overrightarrow{P_iP_j}$ is edge and $P_i < P_j$, we can order the edges by stating $\overrightarrow{P_iP_j} < \overrightarrow{P_lP_k}$ if $P_i < P_l$ or in the case $P_i = P_l$ if $P_j < P_k$.

The n value in the $\kappa(P_1, P_2, \ldots, P_h)$ representation of Q_{gr} depends on the a_i, b_i values of definition (4). It is not difficult to prove that the possible maximum value for n is $n_{max} = 2^k + 2 \cdot k - 2$, while for the simplex

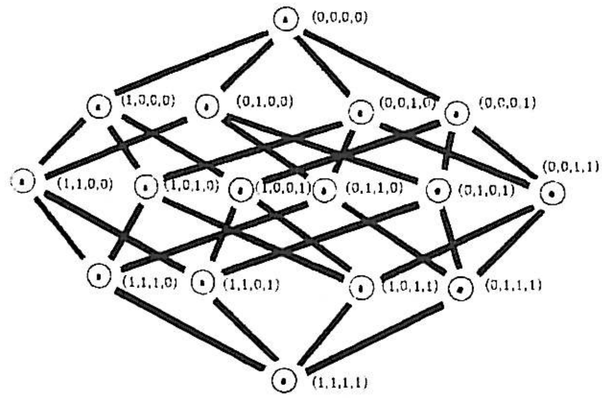

FIG. 2. Edge system of unit cube of four dimensional Euclidian space.

In their study (Mayer and Sibit, 1980) [1] introduced other nonlinear restrictions (constraints) as well. Comparing the technique of the present paper to that of [1], we notice that even thought the method described in this paper can only be applied to linearized restrictions, it is not necessary to introduce a second term in expression (1) which contains subjective parameters.

ALGORITHM OF MINIMIZATION

Our aim is to minimize the quadratic function in expression (1) on the set $Q = \kappa(P_1, P_2, \ldots, P_n)$ which is a convex hull of points $P_i \in \mathbb{R}^k$.

In the three-dimensional space we know that a convex solid is surrounded by two-dimensional polygons, by one dimensional edges and by the zero-dimensional P_i points itself. For the sake of conciseness of description of our algorithm, we state that Q has a three-dimensional surrounding solid as well, namely itself. For example any tetrahedron of $\mathbb{R}^3$ surrounded by the tetrahedron itself, by four triangles, by six edges, and by its four vertices (peak points). This complete system of surrounding configurations we refer as system of boundary hyperplane configurations.

Similar construction can be done in the Euclidian spaces of higher dimension. Basic concepts of this construction are that of the linear algebra and the rank concept of matrices, see e.g. [2]. The exact mathematical construction is reported in [3]. A computer program has been developed to find the system of boundary hyperplane configurations of the $\kappa(P_1, P_2, \dots, P_n) \subset \mathbb{R}^k$ convex solid. We have checked the result of the program for a number of two- and three-dimensional solids, for the cubes and simplexes of higher dimensions, since these are the bodies for which the results are known from theoretical consideration. For example the system of boundary hyperplane configuration of the unit cube of space $\mathbb{R}^4$ consists of the cube itself, 8 pieces three-dimensional (ordinary) cubes, 24 pieces of squares, 32 sections, and 16 peak points. This is the result of the computer program but it can be proved theoretically as well or, in principle, it can be checked directly from Fig. 2.

Knowing the complete system of boundary hyperplane configurations of the convex solid we can describe the algorithm of the minimization. First we note that if matrix A in expression (1) is not singular on the Q set, then the system of boundary hyperplane configuration of Q preserves its structure after the $AQ = \{Ax \mid x \in Q\}$ injection of Q into the space of normalized measurements $\mathbb{R}^m$. Secondly, we know, as the result of convex analysis, that our quadratic function has one and only one minimum value on a convex body, in other words $\hat{x}_Q$ of expression (1) consists of one and only one point.

Now let us study together with each element of the boundary hyperplane configuration the hyperplane of this configuration, i.e. the smaller dimensional translated linear subset of the $\mathbb{R}^m$ space which contains the whole configuration. On this plane we can divide the points of the given configuration into inner and boundary points. Let us state that the zero dimensional P_i peak point's interior is the P_i point itself. Then by finding the interior of every configuration of the AQ convex solid, we create the division of the convex solid AQ into disjoint sets. In this way, from the relation $Ax_Q \in AQ$, it follows that $A\hat{x}_Q$ is an element of the interior of one and only one configuration.

Now, let us study the following algorithm. We project the measured vector $\boldsymbol{b}$ of (1) for all the boundary hyperplane. If the projection, related to the given hyperplane configuration, is outer point or boundary point then we drop this point. Let us denote the set of projections of $\boldsymbol{b}$, if the projected point turned to be inner related to the jth hyperplane configuration by $\{\boldsymbol{b}_j, j \in J\}$, where J is an index set of the configurations. Note that for any zero-dimensional hyperplane the projection of $\boldsymbol{b}$ is P_i, that is an inner point, thus the J set is nonempty, $i \in J$.

Let us suppose that $A \cdot \hat{x}_Q$ belongs to the interior of the configuration enumerated by index $j_0 \in J$. Projecting $\boldsymbol{b}$ to this hyperplane, the projection $\boldsymbol{b}_{j_0}$ is also in the interior of the configuration, otherwise the intersection of the section $[A \cdot \hat{x}_Q, \boldsymbol{b}_{j_0}]$ and the boundary of the j_0th configuration is closer to $\boldsymbol{b}$ than $A \cdot \hat{x}_Q$, which is in contradiction to the definition of $\hat{x}_Q$ of (1). Because $\boldsymbol{b}_{j_0}$ is in the interior of the j_0th configuration, so $A\hat{x}_Q = \boldsymbol{b}_{j_0}$. The j_0 index is not known in advance, but we can choose it easily from the set of those projections where $\boldsymbol{b}_j$ is inner point, getting in this way the solution of minimization problem (1) in the form

$$A \cdot \hat{x}_Q = \{\boldsymbol{b}_j; j \in J \mid \| \boldsymbol{b} - \boldsymbol{b}_j \|^2 \to \min\} \tag{9}$$

A is not singular on the set Q, thus the mapping of Ax allows inversion on the set $\{A \cdot \boldsymbol{x} \mid \boldsymbol{x} \in Q\} \subset \mathbb{R}^m$, therefore knowing point $A\hat{x}_Q$ we easily find $\hat{x}_Q$ itself.

The algorithm above is correct and complete, but we can speed it up considerably by the following additional considerations. If we find that the projection of $\boldsymbol{b}$ for the jth boundary hyperplane configuration is inner, then it is meaningless to project $\boldsymbol{b}$ for any further configuration which is a part of the jth configuration. Because in this case the further projection necessarily gives a higher distance, as the minimum in a subset of a set is not lower (in the given case definitely higher) than the minimum on the set itself. This observation also means that we organize the order of projections not arbitrarily, but going from the higher to the lower dimensions. For example if the first projection of $\boldsymbol{b}$ to the k-dimensional hyperplane of AQ configuration turns to be inner, then we can finish the process, because all the other (lower dimensional) hyperplane configurations are parts of the complete body, consequently this first projection gives the $A \cdot \hat{x}_Q$ point. This will be the more frequent case if the field material and theoretical curves fit each other, especially working with logical restrictions Q_{lr}.

To illustrate the work of the algorithmic steps let us study the situation below designed for simple but substantial geometrical presentation. Let $k = m = 3$ the normalized measured vector $\boldsymbol{b} = (-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4})$, and consider the logical restrictions $Q_{lr} \subset \mathbb{R}^4$ defined by the conditions

$$\begin{aligned} &x_0 + x_1 + x_2 + x_3 = 1, \\ &x_i \geqslant 0, (i = 0, 1, 2, 3) \end{aligned} \tag{10}$$

Projecting Q_{lr} for the $\boldsymbol{x} = (O, x_1, x_2, x_3) = (x_1', x_2', x_3') \subset \mathbb{R}^3$ hyperplane we get Q_{lr}' by the inequations

$$\begin{aligned} &x_1' + x_2' + x_3' \leqslant 1, \\ &x_i' \geqslant 0, (i = 1, 2, 3) \end{aligned} \tag{11}$$

and there is a one-to-one correspondence between Q_{lr} and Q'_{lr}. The inequation system (11) indicates that Q'_{lr} is a tetrahedron, the convex hull of the $P_0=(0,0,0)$; $P_1=(1,0,0)$; $P_2=(0,1,0)$; $P_3=(0,0,1)$ points, $Q'_{lr}=\kappa(P_0,P_1,P_2,P_3)$. Now, for the aim of getting a vivid geometrical picture let us choose matrix A in the form

$$A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{12}$$

which is nonsingular on Q_{lr} and on the $x_0+x_1+x_2+x_3=1$ hyperplane. In this case AQ_{lr} is completely identical with Q'_{lr}, so our task is to find the nearest point of Q'_{lr} to the $b=(-\frac{1}{4},-\frac{1}{4},-\frac{1}{4})$ point, see Fig. 3. We remark that for a less artificial matrix A than that of (12) the character of the minimization process remains similar.

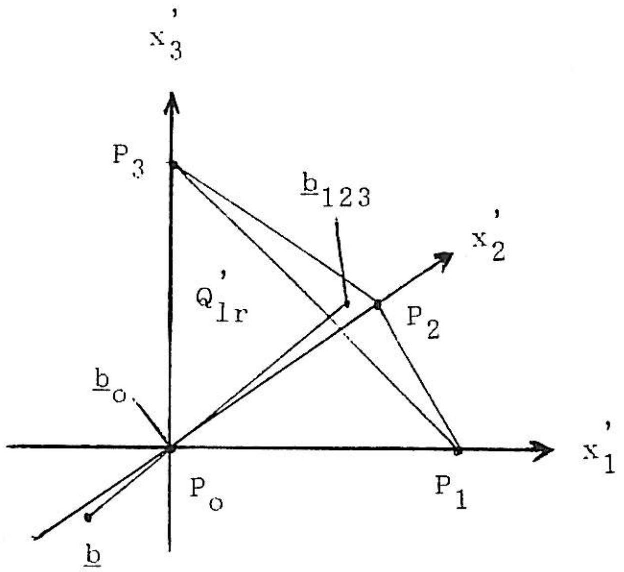

FIG. 3. Geometrical representation of the process of minimization for the case of logical restrictions for three solid components model.

Following our algorithm first we realize, that b is outer point to the tetrahedron $P_0P_1P_2P_3$, thus we have to project b to the four boundary triangles. We found that projection of b is outer to triangles $P_0P_1P_2, P_0P_1P_3, P_0P_2P_3$. Let us denote the projection of b for the $P_1P_2P_3$ triangle by b_{123}. Because of symmetry $b_{123}=(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ which is an inner point of the triangle and $\|b-b_{123}\|=1/\sqrt{3}+\sqrt{3}/4$. Now as was pointed out earlier it is needless to project b for the sections P_1P_2, P_1P_3, P_2P_3 and for the points P_1, P_2, P_3 because these boundary configurations are parts of the triangle $P_1P_2P_3$. There is no more two dimensional configurations, so we project b for the sections P_0P_1, P_0P_2, P_0P_3, and we found again that all these projected points are outer related to the given sections. Thus according to our algorithm we project b to the only remaining vertex P_0 and the result of it is trivially $b_0=P_0=(0,0,0)$ with distance $\|b-b_0\|=\sqrt{3}/4$. In this way the set $\{b_j, j\in J\}$ now looks like $\{b_{123}, b_0\}$, i.e. consists of two elements. Because $\|b-b_0\|<\|b-b_{123}\|$, thus from (9) we get that the minimum point is $A\hat{x}_Q=b_0=(0,0,0)$, from which using (10) and (12) we get the final result $\hat{x}_Q=(1,0,0,0)$.

As to the computer realization of the algorithm we made the following arrangement. We define a table, each line of which is related to one of the boundary hyperplane configurations, and we associate to each line the so called status variable of logical character, which may have the following values: 'U' if the projection is Unexamined yet, 'O' if the result of the projection is Outer, 'I' if the result of the projection is Inner, and 'N' if the realization of the projection is Negligible. Using the scheme presented in Table 1, the realization of the algorithm is the following: As initial value we set the status of each line for 'U'. Then we repeat the only step until there is at least one status 'U' in the table. This step is to find the higher dimensional line in the table with status 'U', then project b to the hyperplane associated to that line and decide whether the projection is Inner or Outer. If the status is 'O' then we drop the result of the projection, if the status is 'I' then we keep it and we set the status of all sub-configurations for 'N'. After completing this loop we choose the minimum distance from the lines with status 'I'. Table 1 is the realization of the algorithm for the geometrical illustration above. The column St_0 denotes the initial status values and actually we made nine projections, thus St_9 is the column where the main loop of our algorithm is ended. In Table 1 we marked with a circle the only two places where we kept the result of projection and we calculated the appropriate distances. Of course for more complicated bodies and for higher dimensions the size of the defined table is extended.

APPLICATION

Let us see the practical consequences of the use of different restriction conditions at the evaluation of volumetric components of a fractured granite rock. The logs involved to the quantitative interpretation are the neutron density, the neutron porosity, the acoustic, the photoelectric index, the thorium and the potassium spectral gamma logs, so in the given case $m=6$. For the sake of accurate porosity estimation we divide the rock matrix for the quartz, the feldspar, the mica and heavy minerals and the pyrite components ($k=4$). The normalizing measure of uncertainty of the logs and the used, slowly changing, parameters

Table 1. Computer Realization of the Process of Minimization for the Case of Fig. 3

$P_0P_1P_2P_3$	Min. value	Min. offset	St_0	St_1	St_2	St_3	St_4	St_5	St_6	St_7	St_8	St_9
1 1 1 1			U	O	O	O	O	O	O	O	O	O
1 1 1 0			U	U	O	O	O	O	O	O	O	O
1 1 0 1			U	U	U	O	O	O	O	O	O	O
1 0 1 1			U	U	U	U	O	O	O	O	O	O
0 1 1 1	$\frac{1}{\sqrt{3}}+\frac{\sqrt{3}}{4}$	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	U	U	U	U	U	(I)	I	I	I	I
1 1 0 0			U	U	U	U	U	U	O	O	O	O
1 0 1 0			U	U	U	U	U	U	U	O	O	O
1 0 0 1			U	U	U	U	U	U	U	U	O	O
0 1 1 0			U	U	U	U	U	N	N	N	N	N
0 0 1 1			U	U	U	U	U	N	N	N	N	N
0 1 0 1			U	U	U	U	U	N	N	N	N	N
1 0 0 0	$\sqrt{3}/4$	(0, 0, 0)	U	U	U	U	U	U	U	U	U	(I)
0 1 0 0			U	U	U	U	U	N	N	N	N	N
0 0 1 0			U	U	U	U	U	N	N	N	N	N
0 0 0 1			U	U	U	U	U	N	N	N	N	N

are close to that published in [4] with exception of the parameters of the pyrite which in the above order of the logs are (4.07 g/cm^3, -0.03 unit, 38 μsec/foot, 15.77 barn/electron, 2.0 ppm, 0.7%).

For completeness we include to our discussion the following heuristic approximate solution of the problem of logical restrictions (1) and (3) which, as far as we know, is incorporated in many log analyzing programs. This solution consists of two steps. First we calculate as a result of one simple projection the vector solution $\hat{x}_{Q_{nr}}=(x_0, x_1, \ldots, x_k)$ of the problem (1) and (2), then we modify this vector in the following way

$$x_j' = \frac{\max\{0, x_j\}}{\sum_{i=0}^{k} \max\{0, x_i\}}, (j=0, 1, \ldots, k) \tag{13}$$

which means we round the negative components X_i of $\hat{x}_{Q_{nr}}$ for zero and we divide the other components by the sum of positive components in order to fulfill the material balance equation. It is clear that

Table 2. Results of Interpretations under Different Restriction Conditions at a Massive Granite Zone (* denotes that the given value is a boundary value)

9381.	ϕt	Qz	Fp	M&Hm	Pr	INC
Qnr	.0380	.4947	.4359	.0437	−.0123	.309
Pℓr	.0376	.4887	.4306	.0432	.0000*	.458
Qℓr	.0408	.4741	.4577	.0274	.0000*	.399
Qgr	.0452	.4407	.4932	.0009	.0200*	.731

Table 3. Results of Interpretations under Different Restriction Conditions at a Fractured Granite Zone

9397.	ϕt	Qz	Fp	M&Hm	Pr	INC
Qnr	.1292	.7610	−.1480	.1883	.0695	1.55
Pℓr	.1125	.6629	.0000*	.1640	.0606	2.12
Qℓr	.1371	.6507	.0000*	.1223	.0898	1.75
Qgr	.1400	.6062	.0600*	.0959	.0978	1.92

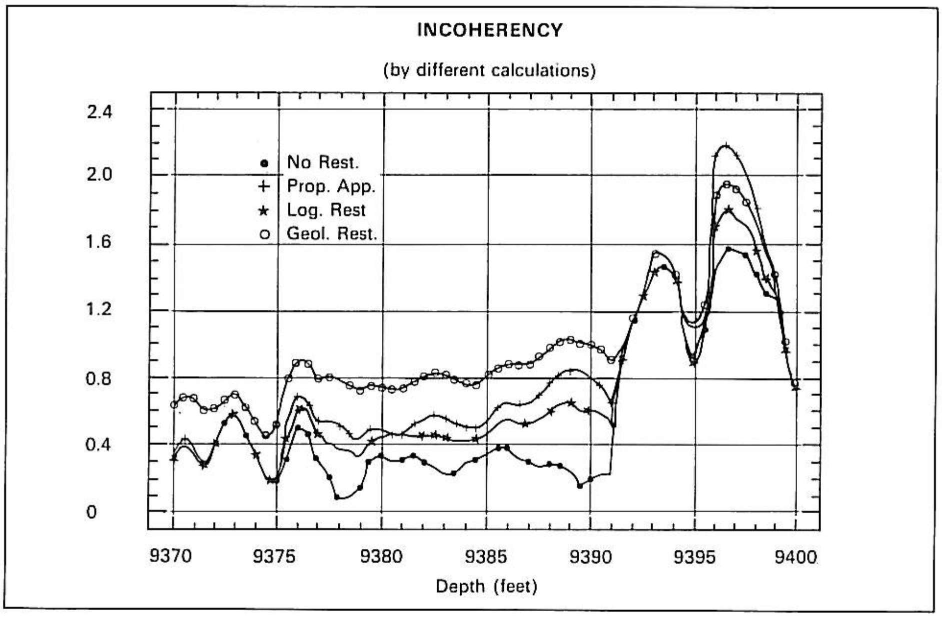

FIG. 4. Incoherency as a function of depth between 9370 and 9400 feet.

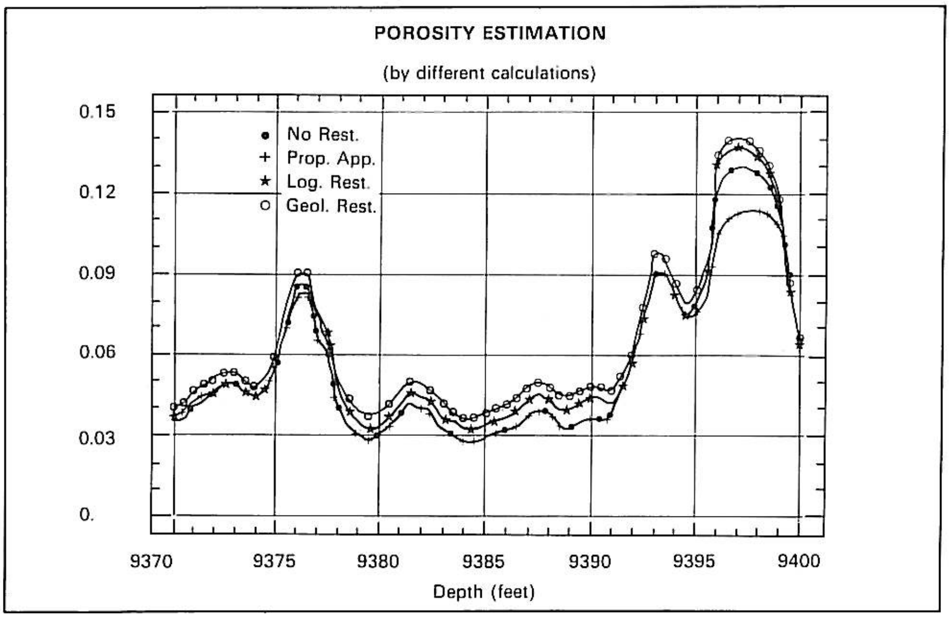

FIG. 5. Porosity estimations on the log section of Fig. 4.

$(x'_0, x'_1, \ldots, x'_k) \in Q_{lr}$, so we denote the modified vector by $x_{P_{lr}}$ and we call it as proportional approximation of problem (1) and (3).

Let us study the result of different interpretations in the granite rock, first at the depth 9381 feet (Table 2). Evidently the no restriction case gives the smallest incoherency $\| b - Ax \|$, but because of the negative pyrite content, $\hat{x}_{Q_{nr}}$ could not be accepted at all, in other words the probability of occurrence of the rock components $\hat{x}_{Q_{nr}}$ is zero. According to formula (13) the modification of $\hat{x}_{Q_{nr}}$ which is the proportional approximation $\hat{x}_{P_{lr}}$ eliminates the negative pyrite content, but necessarily gives a higher incoherency than the exact solution $\hat{x}_{Q_{lr}}$.

The change in the main porosity component is not significant and the interpretation based on any of the four curves in Fig. 5 indicate that we are in the massive granite zone.

Table 3 shows the results at the depth 9397 feet. Here we have a higher negative feldspar component. Because the sum of the positive components is more than one, and because practically we shall not find negative porosity estimation in the solution of the Q_{nr} task, thus the porosity of the proportional approximation is less than the original one. On the other hand the exact porosity estimation offered by $\hat{x}_{Q_{lr}}$ is found to be higher in the present example than that of $\hat{x}_{Q_{nr}}$. This means that the proportional approximation results could not be considered as a good approximation of the Q_{lr} task (1) and (3). This statement becomes especially clear having in mind that the modifying step is not a function of the matrix A of (1).

Figure 4 shows the incoherency of different calculations on the depth interval 9370–9400 feet. We can see that, with the exception of those intervals where the Q_{lr} restrictions fulfill automatically (e.g. 9370–9375 feet) and consequently $\hat{x}_{Q_{nr}} = x_{P_{lr}} = \hat{x}_{Q_{lr}}$, the decrease in incoherency of $\hat{x}_{Q_{lr}}$ related to $x_{P_{lr}}$ is considerable. In Fig. 5 we can follow the porosity estimations obtained by different calculations. Despite of the considerable changes in incoherency, the porosity shows a relatively stable character, especially in the massive granite intervals. This emphasises again that the porosity is not only the target parameter of the complex well log interpretation but the most robust one. However difference between the porosity estimations of Q_{lr} and P_{lr} is not negligible in the fractured zones and this fact tells about the usefulness of the exact problem solution.

In the case when we have additional information about mineral contents, then we solve the geological restriction task (1) and (4). For example if we know that pyrite content is more than 2% and feldspar content is at least 6%, i.e. $a_5 = 0.02$ and $a_3 = 0.06$, while all other limits are zero and one, then we get the results shown in Tables 2 and 3 and the appropriate curves of Figs. 4 and 5. The incoherency belonging to the solution $\hat{x}_{Q_{gr}}$ is necessarily higher (or equal) than that of the $\hat{x}_{Q_{lr}}$. But if this increase is not so dramatic (this is the case at 9397 or 9393 feet), then we should accept this solution, which is in accordance with the geological knowledge of the given area.

CONCLUSION

- Restriction conditions of well log interpretation can be represented or approximated in the form of defining the domain of minimization as a convex solid of a higher dimensional space.
- Rather exact algorithm for the minimization of a quadratic function on a convex solid can be worked out on the basis of elementary projections of different dimensions.
- Application of the exact algorithm reduces the incoherence between measured and theoretical logs and in some fractured zones of granite rock effects for the porosity estimation are considerable.

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