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Estimation of the Error Range of True Vertical Depth Determination

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تقدير مدى الخطأ في تحديد العمق العمودي الحقيقي

غابور سيغاتي

تقدم هذه الورقة نموذجاً لمنحنى من النوع الحلزوني ببارمترين، لوصف الوضع الهندسي المحتمل لعملية الحفر الدوار العميق، حيث تم التعرف على الأسباب التي تحول دون اتخاذ ثقب البئر لشكل الخط المستقيم وتصنيفها، وفي هذا الإطار تم استخدام طرق الهندسة التفاضلية لتحليل الطول، والانحراف الزاوي للمنحنى عن العمودي، والانحناء الجاوسي للنموذج، كما تم تفسير الانحناء بطريقة تخص ثقوب الآبار المبطنة، وقد أعطى التحليل العددي تقديراً لخطأ العمق العمودي عبارة عن دالة في بارمترين مستقلين أو دالة في الانحراف الزاوي للثقب. أهمية هذا التقدير تتعلق بالوضع الهندسي لتماس النفط بالماء الذي يمثل بدوره مؤشراً على نوعية الخزان النفطي.

Abstract: *The present paper suggests a spiral type curve model with two parameters for potential description of the geometry of deep rotary drilling. The reasons that borehole geometry never coincides with a straight line are enumerated and classified. The length, the angular deviation of the curve from vertical, and the Gaussian curvature of the model are analyzed with the methods of differential geometry. The curvature is interpreted in a way specific to a cased hole. Numerical analysis gives an estimation of true vertical depth error as a function of the two independent parameters and as a function of angular deviation of the hole. The importance of this estimation is related to knowledge of the geometry of the oil-water contact that also serves as reservoir quality indicator.*

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INTRODUCTION

Causes of Tilted Hole at Deep Rotary Drilling and their Classifications

Even when borehole design (construction) describes vertical deep well for oil or other industry, the actual borehole shaft will differ from the perfect vertical. Full scale modeling of the hole can be considered as a curve, starting at the point of the collar (mouth) of the well and finishing at the point of the bottom of the hole (shaft). A perfect vertical hole would mean a vertical straight line from the mouth of the well to the bottom of the well. In this case, the true vertical depth (TVD) and the length of the hole are in exact coincidence. In general, the TVD is the distance between two horizontal planes each of them intersecting one of the mentioned points. The actual hole is a real, three dimensional curve between the two points. The deviation of this curve from a straight line is the result of a number of technical, technological causes, on one hand, and of geological causes on the other hand.

The first group of causes are described[1]. Geological causes occur when the boring tool pierces rocks of changing hardness or tectonic dislocations. Among other important causes are fault zones, layer boundaries, boundaries of concretions, voids, karts and brachiated zones. In these cases, the tool tends to traverse along lines of least resistance. Accordingly, at layer boundaries there is a tendency for the borehole to tilt towards the normal direction of the bedding plane. As another example, there is an increased chance for having stronger deviations when the hole axis forms a relatively small angle (5^0 - 30^0) with a steeple dipping hard rock. In these cases, both the azimuth and the zenith angles of the borehole inclination may change.

Rich enough material can be found to prove that by time and by development of drilling technology, the technical-technological causes of unwanted deviations have sharply decreased[2]. In contrast, some of the geological causes of deviations are difficult to compensate by any sophisticated technology.

The geological causes can be examined in terms of their random or systematic character. This can be done from the perspective of whether or not deviation of two neighboring holes will be similar. The general gentle slope of the basin can be expected to cause systematic deviation, while voids and karts zones are likely to cause random deviations. However, the presence of faults is likely to affect the variation in such a way that the deviations obtained may have both random and systematic elements at the same time.

Importance of the Problem and Possible Different Approaches to the Solution

The problem of an improved knowledge about borehole geometry becomes important in some cases. One such case is when we have a number of wells crossing hydrocarbon (HC) reservoir and reaching the water body below the reservoir. In this case, the exact spatial position of the oil water contact (OWC) and the capillary transition zone (CTZ) has a high importance as reservoir rock quality indicator.

Indeed, the closer this surface is to a horizontal plane , the better the quality of the reservoir rock. This reservoir rock quality indicator expresses the global, overall quality of the reservoir rock in contrast to core samples having only local validity. Core drilled rock material represents always only a tiny fraction of the total reservoir volume.

When an inclination survey has been done in the hole, the spatial position of the hole can be restored exactly. Frequently, this survey data is not available. In this case, knowledge of the magnitude of the potential true vertical depth error, as a function of the deviations is of interest. Such relations can be obtained only by modeling the global geometry of the hole.

MODEL OF THE DRILLED HOLE ABD ITS MAJOR PARAMETERS

Spiral Type Curve Model of the Hole Geometry

As the drill bit moves downward, the probability that the projection of the actual bottom of the hole to the horizontal surface deviates from the mouth of the well increases. Here, the geometry of the hole is modeled in a deterministic, rather then in a stochastic way. From this, a model is suggested, whereby the side distance increases linearly with depth. This requirement prescribes that the hole model curve runs on the surface of a (narrow) cone. Also, from the point of view of this deterministic behavior, a continuous, analytical type of changes in the shape of the model curve is considered. In this way, the following definition for the hole model curve is obtained.

To define a three-dimensional curve in Cartesian coordinates is to define the $r(t) = (x(t);y(t);z(t))$ vector-scalar function.

For the spiral-type model curve of the hole, $r(t)$ is given by the following equations:

$$x(t) = at \cdot \cos(bt)$$
$$y(t) = at \cdot \sin(bt) \qquad (1)$$
$$z(t) = t$$

In this curve definition, a special three-dimensional spiral-type of curve is described, which contains two parameters a and b. Parameter a describes the cone, the surface of which contains the curve. Indeed, for any point of the curve, for any t value,

$$\mathbf{r}_{xy}^2(t) = x^2(t) + y^2(t) = (a \cdot t)^2$$

i.e. the distance of the curve from the z-axis is increasing by t linearly and the coefficient of the grow (speed) of radius r_{xy} is given by parameter a. This cone is refered to as the supporting cone of curve (1). The curve is somewhat similar to a helix curve but the helix curve is contained by a cylinder, rather than by a cone[3].

The geometry of the model curve is normalized, by selecting for the independent t variable of the curve the limits.

$$0 \le t \le 1 \qquad (2)$$

This places the top of the hole at the beginning of the coordinate system and the bottom of the hole to the a radius circle around the point (0,0,1) with a z-direction axis. This means from the borehole application problem point of view that TVD = 1, therefore the length unit is TVD itself. One of the major tasks is to calculate the length of the curve. The starting and finishing points of the curve given by (2), give the parameter b a clear meaning. Measuring the angle in radians and introducing the definition $b = 2\pi \cdot f$, f describes the number of rotations of the curve until it reaches the bottom level. This number is not necessarily large, and values below $^1/_2$ for f ($f \leq .5$) allow the examination of a hole that has a systematic deviation direction in terms of geography (*e.g.* to the North). Figure 1 is used to demonstrate the suggested geometrical model of the hole. Here, the Cartesian coordinate system is introduced in an unusual position, but it keeps the right hand rule. This is to avoid a well sketch 'deepening' upward while using vector products of vectors, as below, without changing the right hand rule. In figure 1, both parameters a and b are exaggerated from values that are relevant for the borehole model for the purpose of representing their graphical meanings clearly.

Length, Deviation Values and Curvature of the Spiral Type Model of the Hole

To calculate the length $L_{ab}(0,1)$ of curve (1) of variable t, one needs to know the $\left\|\frac{d\mathbf{r}}{dt}\right\|$ length element, for which

$$\left\|\frac{d\mathbf{r}}{dt}\right\|^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 \quad (3)$$

relation is valid. After applying equation (3) to curve (1) *i.e.* making the calculations for the derivatives and using the basic properties of the trigonometric functions; the elementary lengths are integrated in

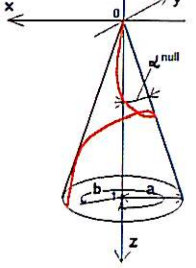

Fig. 1. Curve model for the geometery of the drilled hole.

interval (2). In this way, the length of the curve (1), $L_{ab}(0,1)$ is obtained as

$$L_{a,b}(0,1) = \int_0^1 \sqrt{(1+a^2) + a^2b^2 \cdot t^2}\, dt \quad (4)$$

integral in equation (4) for any a, b parameters can be expressed in a closed form by primitive function only in a complex way and its numerical calculation is more feasible.

The borehole inclination angle, α in the $\mathbf{r}(t)$ point of the curve can be calculated from the scalar products of vector $\frac{d\mathbf{r}}{dt}$ and z-direction unit vector (0,0,1). On this basis, the cosine of the inclination (zenith) angle α, is calculated from equation (5).

$$\cos(\alpha) = \frac{1}{\left\|\frac{d\mathbf{r}}{dt}\right\|} = \frac{1}{\sqrt{(1+a^2) + a^2b^2 \cdot t^2}} \quad (5)$$

α is a monotone increasing function of t. Consequently, the smallest inclination angle α is at $t = 0$, where this angle is the basic (half) angle of the supporting cone (see Fig 1.). The angle of the supporting cone can be denoted as α^{null}. For this angle, $tg(\alpha^{null}) = a$ and this equation is equivalent to that of equation (5) for the case $t = 0$. For any other $t > 0$ the $\alpha > \alpha^{null}$ relation is valid, and $\alpha^{\max}$ is reached in equation (5) at $t = 1$.

From the point of view of hole geometry, a further property of curve given by equations (1) is its Gaussian curvature, which is the local limit of angular (directional) change of the tangent vector of the curve with respect to the length of the arc. For the spatial curve $\mathbf{r}(t)$ the Gaussian scalar curvature, $\kappa(t)$ can be calculated on the basis of the formula [4]

$$\kappa(t) = \frac{\left\|[\mathbf{r}'(t), \mathbf{r}''(t)]\right\|}{\left\|\mathbf{r}'(t)\right\|^3} \quad (6)$$

where [,] denotes the vector products of the two vectors in brackets.

For our curve, on the basis of the rule for the derivation of multiplication of functions, the first and second derivatives are

$$\mathbf{r}'(t) = (a \cdot \cos(bt) - t \cdot ab \cdot \sin(bt)\,; \\ a \cdot \sin(bt) + t \cdot ab \cdot \cos(bt)\,; 1) \quad (7)$$

$$\mathbf{r}''(t) = (-2ab \cdot \sin(bt) - t \cdot ab^2 \cdot \cos(bt)\,; \\ 2ab \cdot \cos(bt) - t \cdot ab^2 \cdot \sin(bt)\,; 0) \quad (8)$$

Substituting equations (7) and (8) into equation (6) and after tedious but elementary calculations the curvature of spiral –type hole model is given as:

$$\kappa(t)=ab\cdot\frac{\sqrt{(4+4a^2)+(1+4a^2)\cdot b^2t^2+a^2b^4\cdot t^4}}{[(1+a^2)+a^2b^2.t^2]^{3/2}} \quad (9)$$

ANALYSIS OF THE CHARACTERISTICS OF THE HOLE MODEL CURVE

The importance of the curvature described by equation (9) is that while the hole inclination relates the actual direction of the hole to an external, vertical direction, the curvature describes the rate of directional change from a previous section of the hole.

In our situation for the cased hole, a special interpretation of this curvature is suggested, which corresponds to real drilling activity, as follows. Consider that the hole consists of short, exact straight sections (intervals) of uniform length jointed to each other. This arises from the assumption, that the casing tubes remain straight in the hole, and that the casing string adjusts its shape to the changing hole geometry by inclinations at the jointing of the casing tubes. Figure 2 shows the situation as described. The common length of the casing tube is denoted by p. The hole geometry is locally approximated by a tangent to the curve circle, in the plane of vectors $\mathbf{r}'(t),\mathbf{r}''(t)$, and of radius $\frac{1}{\kappa(t)}$. Here, the fact that the curvature of a section of a circle is the reciprocal of the radius of this circle is used[5]. The end points of the p length intervals are supposed to lie on this approximating circle. For the angle b what is the angle of inclination of two jointing casing tubes, using the regular polygon geometry on Fig. 2, the tangent of β/2 can be expressed as a ratio of β/2 and $\frac{1}{\kappa(t)}$. For real holes, one can not expect sharp directional changes, consequently the curvature is small and therefore; $p<<\frac{1}{\kappa(t)}$(unlike that shown from the exaggerated Fig. 2) and *tg* (β/2) is close to β/2.

In this way one derives, as a special interpretation of the curvature to the cased hole, the relation:

$$\beta = p\cdot\kappa(t) \quad (10)$$

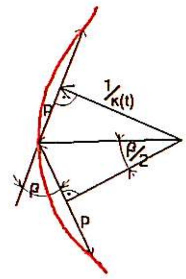

Fig. 2. Interpretation for curvature for cased hole.

As a starting point for analysis of formula (9) for the curvature, it is worthwhile to study it without the limits in equation (2). For large t tending to infinity, $k(t)$ tends to $\frac{1}{a\cdot t}$. This means that the larger the variable t, the smaller the effect of parameter b, and finally, the limit of the curvature is determined by the horizontal cross section of the supporting cone. For the case $t=0$ however, as direct calculation shows, parameter b has a large effect on the curvature, because here $\kappa(0)=\frac{2ab}{1+a^2}$.

For the case with the limits given by equation (2) for the model of the hole, the major feature of the curvature derives from the fact that for a vertical hole the deviation of the bottom of the hole from the top of the well is much smaller than the depth of the well. Expressing it with the introduced quantities, $a<<1$. Because t is limited and also b is not large in equation (9), all the terms with the multiplier a are negligible. Therefore, the denominator of equation (9) becomes 1 and one obtains for $a<<1$:

$$\kappa(t)=ab\cdot\sqrt{4+b^2t^2} \quad (11)$$

The largest curvature, and therefore according to equation (10), the largest joint deviation angle β is given by formula (11) at $t=1$.

For a real borehole A1-52 well of the Ghadames Basin of Libya where complete borehole deviation survey is available, the following data are known. The logging depth of the bottom of the hole (BHT) is 9567 feet, while side deviation of BHT is found to be 158 feet to the direction given by the azimuth angle 157^0 of SSE. This corresponds to a $\alpha^{null}=.94^0$.

It should be noted that this direction is almost perpendicular to the direction of the closure of outcrop on the geological map of Libya, proving that in the Ghadames Basin which has a quiet and regular structure of sedimentation the borehole is apt to deviate towards the normal of the bedding planes.

On the basis of the presented model and formulas, calculations can be made, whereby the possible length and borehole deviation parameters are calculated. They are calculated as a function of known or assumed major, independent parameters of the model curve of borehole a and b or what is almost the same a and f ($b = 2\pi \cdot f$). The integral in equation (4) is calculated by eight point Gaussian quadrature process, with automated selection of subintervals for integration, which provides the necessary high accuracy[6] in the range of 10^{-6}. The calculations are made by appropriate source code developed by the author and named Hole Geometry for using Fortran language[7] .

Table 1 contains the results of the calculations for the independent parameter set of a and f, which are in the range of a typical successfully drilled vertical hole. The exact meaning of the two input columns (Ic1, Ic2) as well as that of the four output columns (Oc1, Oc2, Oc3, Oc4) are written in the nomenclature type attachment belonging to the Table. In the Table all the angles are expressed in degrees (while in all the formulas of this paper angles should be used in radians), and angle β is calculated with the parameter $p = {}^{1}/_{50}$.

The main column Oc1 is calculated from formula (4) in the above mentioned way and gives the length of the hole, which can be identified by the logging depth expressed in the introduced length unit (TVD = 1) rather than in the more regular unit feet. For the parameter range $a << 1$ in the case of the absence of the Fortran program, the following useful approximations can be derived. From formula (5) for $t = 1$, expressing $tg(\alpha)$ by $\cos(\alpha)$ one gets, . For $t = 0$,

$$\alpha^{max} = a \cdot \sqrt{1+b^2}$$

, while from formulas

$$\alpha^{null} = arc\ tg(\alpha^{null}) = a$$

(10) and (11) $\beta_{loc}^{max} = p \cdot ab \cdot \sqrt{4+b^2}$.

Column Oc4 accounts for the TVD error of a well of 10000 feet TVD. The calculated TVD errors are small for the hole of a slight deviation of $1^0, 2^0, 3^0$ degree compared to the TVD of the hole. For example for $\alpha^{null} = 2^0$ and $f = .5$, i.e. for a hole that still has a general, oriented deviation, the error is 26 feet, which is small. This error is far from being negligible when the much more delicate task to determine the geometry of OWC is considered.

CONCLUSIONS

1. Assuming small angular deviation of the hole, the true vertical depth determination error is not negligible from the point of view of such a

Table 1. Calculated values of the hole model and description of their meaning

Ic1	Ic2	Oc1	Oc2	Oc3	Oc4
1.0	0.00	1.00015	1.0	0.0	2
1.0	0.25	1.00028	1.9	0.1	3
1.0	0.50	1.00065	3.3	0.2	7
1.0	0.75	1.000128	4.8	0.5	13
1.0	1.00	1.00215	6.3	0.8	22
1.0	2.00	1.00811	12.3	3.2	81
2.0	0.00	1.00061	2.0	0.0	6
2.0	0.25	1.00111	3.7	0.2	11
2.0	0.50	1.00261	6.6	0.5	26
2.0	0.75	0.00510	9.5	1.0	51
2.0	1.00	1.00857	12.4	1.7	86
3.0	0.00	1.00137	3.0	0.0	14
3.0	0.25	1.00250	506	0.2	25
3.0	0.50	1.00587	9.8	0.7	59
3.0	0.75	1.01143	14.0	1.4	114

Ic = Input column
Oc = Output column

Ic1	angle	α^{null}	°, degree	Angle of the supporting cone. $tg(\alpha^{null}) = a$
Ic2	number f		rotation hole	Rotation number of the around its axes. $b = 2\pi \cdot f$
Oc1	length	$L_{ab}(0,1)$	unit	Length of the hole, while true vertical depth of the bottom of the hole is 1 length unit (formula (4))
Oc2	angle	α^{max}	°, degree	Maximum deviation of the hole from vertical (formula (5) at $t = 1$)
Oc3	angle	β_{loc}^{max}	°, degree	Maximum angle of casing joints, calculated from the maximum hole curvature with the casing length parameter, $p = {}^{1}/_{50}$ (formula (9) and (10) at $t = 1$)
Oc4	abs.err.	Δz	feet	Absolute error of true vertical depth determination for a well of 10.000 feet

demanding task like OWC determination and reservoir scale reservoir rock characterization.

2. True vertical depth determination can be fully established only by using borehole deviation survey.
3. In case of having well established geological, petrophysical criteria, this criteria should be enhanced over seemingly exact depth data.

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