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Modified Approach to Calculate Water Influx (Unsteady State)

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طريقة معدلة لحساب تدفق الماء غير المستقر

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تم في هذه الورقة وباستعمال طرق مختلفة[4,3,2] حل مثالين لحساب تدفق الماء غير المستقر من كتاب أساسيات الهندسة النفطية[1] بهدف توضيح أن معادلات Marsal مازالت في حاجة إلى بعض التحوير في الثوابت حتى تتوافق مع الحل الأصلي لـ Van Everdingen وHurst. إن هذه التعديلات غير ثابتة ولكنها تعتمد على المدى لقيم t_{ED}، t_D.

Abstract: *In this paper two examples for the calculation of water influx from Ref [1] are resolved using different techniques (Van Everdingen and Hurst[2], Fetkovich[3], Marsal[4] in order to demonstrate that Marsal Equations need some modification on the constants in order to be more consistent with the original solution of Van Everdingen and Hurst[2]. These modifications are not constant but are function of the range of* r_{eD} *and* t_D.

INTRODUCTION

A great number of papers were written about the subject of water influx calculations in unsteady state for radial and linear flow; so there is no need to go in detail about the theory of this subject .

The unsteady state influx theory of Van Everdingen and Hurst is used to calculate the cumulative water influx, under practically all circumstances, for radial and linear aquifers. Unfortunately, it has the disadvantage that the calculations are rather tedious due to the complexity of superposing solutions for each time step, also it requires tables of We_D and t_D.

Fetkovich tried to avoid or remove the necessity of superposition in theVan Everdingen and Hurst method .However it is used for limited aquifers only. Also it is initially still necessary to apply the unsteady state influx theory of Van Everdingen and Hurst for the first few time steps.

Marsal has shown that the Van Everdingen and Hurst tables of We_D and t_D can be fairly well approximated by the infinite radial unit function up to a dimensionless time of $t_D = 0.4\,(r_{eD} - 1)^2$. He used analytical expression to calculate the dimensionless water influx (W_{eD}).

The modified approach used almost the same analytical expression of Marsal but with slight modifications on the constants and there are some constrains on the values of r_{eD} and t_D.

THE MODIFIED APPROACH

Marsal has shown that the Van Everdingen and Hurst tables of We_D and t_D can be fairly well approximated by the infinite radial unit function up to a dimensionless time of $t_D = 0.4\,(r_{eD} - 1)^2$ ....(1)

The approximating function can be written in the following form:

$W_{eD} = 0.5\,(r_{eD}^2 - 1)\,[1 - \exp(-2\,t_D / j^*)]$..........(2)

Where :

$j^* = r_{eD}^4\,\ln(r_{eD}) / (r_{eD}^2 - 1) + 0.25\,(1 - 3\,r_{eD}^2)$ ....(3)

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$t_D = 2.309kt / \Phi\mu_w c_t r_o^2$ ....(4)

$r_{eD} = r_e / r_o$ ....(5)

The dimensionless time and aquifer constant for finite radial aquifers are defined as for infinite radial aquifers.

Applications of the above equations on the examples in Ref [1] gave some deviation from the results obtained using method of Van Ever and Hurst, especially for large value of re_D. So trial and error was used on the constants in the above equations and some constrains were put on the values of re_D and t_D as follow:

For $r_{eD} <= 6$ then

$j^* = r_{eD}^4 \ln(r_{eD}) / (r_{eD}^2 - 1) + 0.245 (1 - 3 r_{eD}^2)$ ......(6)

For $r_{eD} > 6$ then

$j^* = r_{eD}^4 \ln(r_{eD}) / (r_{eD}^2 - 1) + 0.3071 (1 - 3 r_{eD}^2)$.....(7)

For $t_D <= 10$ then

$W_{eD} = 0.5 (r_{eD}^2 - 1) [1 - \exp(-2.2 t_D / j^*)]$....... (8)

For $10 < t_D <= 20$ then

$W_{eD} = 0.5 (r_{eD}^2 - 1) [1 - \exp(-2.15 t_D / j^*)]$.......(9)

For $t_D > 20$ then

Table 1. Input data

Time (years)	t_D	Pressureat o.w.c (psia)	$We_D(r_{eD} = 10)$	$We_D(r_{eD} = 5)$
0	0.00	2720 (p_i)	0.00	0.00
1	5.67	2500	4.95	4.88
2	11.34	2290	8.12	7.46
3	17.01	2109	10.90	9.10
4	22.68	1949	13.50	10.09
5	28.35	1818	15.90	10.83
6	34.02	1702	18.10	11.27
7	39.69	1608	20.20	11.52
8	45.36	1535	22.20	11.69
9	51.03	1480	24.00	11.81
10	56.70	1440	25.70	11.89
h = 100 ft, k = 200 md, r_o = 9200 ft., c_w = 3.x 10^{-6} psi^{-1}, c_f = 4.x10^{-6} psi^{-1}, f = 140° / 360°, Φ = 0.25, μ_w = 0.55 cp.				

$W_{eD} = 0.5 (r_{eD}^2 - 1) [1 - \exp(-1.875 t_D / j^*)]$.....(10)

Examples

Two examples were taken from reference [1]. The input data are shown in Table 1. The four methods were applied using a short computer program (available on request) to calculate the cumulative water influx .The results are shown in Tables 2 and 3.

Table 2. Results of cumulative water influx calculated by different methods x10^6bbl r_{eD} = 5

Time Year	t_D	Van and Hurst	Fekvich	Marsal	Modified
1	5.67	3.77	3.93	3.56	3.78
2	11.34	12.84	13.55	12.43	13.05
3	17.01	24.09	25.51	23.71	24.51
4	22.68	35.67	37.98	35.72	36.46
5	28.35	47.25	49.95	47.43	47.70
6	34.02	58.02	60.88	58.33	58.25
7	39.69	67.76	70.64	68.15	67.84
8	45.36	76.23	79.00	76.68	76.24
9	51.03	83.36	85.93	83.84	83.32
10	56.70	89.21	91.47	89.77	89.11

Table 3. Results of cumulative water influx calculated by different methods x10^6bbl re_D = 10

Tim Year	t_D	Van and Hurst	Fekvich	Marsal	Modified
1	5.67	3.82	2.72	2.65	3.25
2	11.34	13.46	10.35	10.09	12.20
3	17.01	26.44	21.88	21.35	25.46
4	22.68	41.92	36.46	35.59	40.94
5	28.35	59.10	53.31	52.07	57.61
6	34.02	77.46	71.75	70.13	76.00
7	39.69	96.66	91.27	89.92	95.40
8	45.36	116.14	111.30	108.92	115.37
9	51.03	135.46	131.35	128.62	135.37
10	56.70	154.29	151.06	148.02	154.98

CONCLUSION

The results obtained using the suggested modifications of the Marsal Equations indicate clearly that the the Marsal constants need to be varied based on the values of r_{eD} and t_D .It is suggested that the Marsal Equations be re –written in the following form:

$$J^* = r_{eD}^{4} \ln(r_{eD}) / (r_{eD}^{2}-1) + C_1 (1 - 3r_{eD}^{2})$$

$$W_{eD} = 0.5 (r_{eD}^{2} - 1) (1 - Exp(C_2 t_D / J^*))$$

Due to the limited data available for analysis it was not possible to develop an analytical relationship between C_1 and J^* or between C_2 and t_D.

The main objective of this paper was to show the importance of varying the C_1 and C_2 constants based on the values of r_{eD} and t_D to conform with original solutions of Van Everdingen and Hurst. Consequently, it is recommended that future work should be devoted toward development of such analytical relations.

Nomenclature

c_f	=	aquifer compressibility, psi^{-1}
c_w	=	water compressibility, psi^{-1}
c_t	=	$c_f + c_w$
h	=	aquifer thickness, ft
k	=	permeability, md
r_o	=	reservoir radius, ft
r_e	=	aquifer radius, ft
r_{eD}	=	dimensionless radius
t	=	time, year
t_D	=	dimensionless time
Φ	=	porosity, fraction
μ_w	=	water viscosity, cp

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